

Contents

Preface — v

Sarah E. Gasda, Elsa du Plessis, and Helge K. Dahle

Upscaled models for CO₂ injection and migration in geological systems — 1

- 1 Introduction — 1
- 2 Background — 3
- 3 Model description — 8
- 3.1 Key assumptions and dimensionless groupings — 9
- 3.2 Vertical fluid and pressure distribution — 12
- 3.3 Model derivation — 14
- 3.4 Upscaling and subscale processes — 17
- 4 Model application — 24
- 5 Summary — 32

Markus Wolff, Yufei Cao, Bernd Flemisch, Rainer Helmig, and Barbara Wohlmuth

Multipoint flux approximation L-method in 3D: numerical convergence and application to two-phase flow through porous media — 39

- 1 Introduction — 40
- 2 The MPFA L-method in 3D — 41
- 2.1 Details of the scheme — 42
- 2.2 Criterion for choosing the proper L-stencil — 46
- 2.3 Boundary handling — 47
- 3 Numerical convergence — 48
- 3.1 Benchmark test 1 — 50
- 3.2 Benchmark test 3 — 56
- 4 Grid adaptivity — 60
- 4.1 Boundary handling — 62
- 5 Two-phase flow applications — 63
- 5.1 Two-phase model description — 64
- 5.2 Buckley–Leverett-type problem — 66
- 5.3 McWhorter-type problem — 67
- 5.4 DNAPL infiltration problem — 68
- 5.5 Refinement and coarsening indicator — 73
- 6 Summary and conclusions — 74

Alain P. Bourgeat, Sylvie Granet, and Farid Smäi

Compositional two-phase flow in saturated–unsaturated porous media: benchmarks for phase appearance/disappearance — 81

- 1 Introduction — 82
- 2 Definition and basic assumptions — 83

3	Equations —	84
3.1	Mass conservation of each component —	84
4	Choice of the primary variables —	85
5	Presentation of the two test cases —	88
5.1	First test case: gas phase appearing/disappearing by gas injection in a water-saturated rock core sample —	88
5.2	Second test case: evolution from an initial out of equilibrium state to a stabilized stationary one, in a sealed porous core sample —	93
6	Conclusions and perspectives —	104

Marco Discacciati

Coupling free and porous-media flows: models and numerical approximation — 107

1	Introduction —	107
2	Setting of the problem —	108
2.1	The surface-groundwater flow problem —	109
2.2	Interface conditions to couple surface and groundwater flows —	111
3	Weak formulation and analysis —	113
3.1	Mixed formulation of Darcy's equation —	116
3.2	Time-dependent Stokes/Darcy model —	117
4	Multidomain formulation of the coupled problem —	118
4.1	The Stokes/Darcy problem —	119
4.2	The Navier–Stokes/Darcy problem —	122
4.3	Well-posedness of the interface problems —	123
5	Finite element approximation of free and porous-media flows —	124
5.1	Galerkin finite-element approximation of the Stokes/Darcy problem —	126
6	Algebraic formulation of the linear interface problem and solution techniques —	127
6.1	Numerical results —	129
6.2	Other preconditioning methods —	130
7	Iterative methods for the Navier–Stokes/Darcy problem —	131
8	Subdomain iterative methods for the time-dependent (Navier–)Stokes/Darcy problem —	134

Jürgen Fuhrmann

Mathematical and numerical modeling of flow, transport, and reactions in porous structures of electrochemical devices — 139

1	Introduction —	139
1.1	Model scales —	140
1.2	The direct methanol fuel cell – an example of an electrochemical device with a porous electrode —	140

2	Electrolytes and interfaces —	141
2.1	Dilute electrolytes —	141
2.2	Bulk electroneutrality —	143
2.3	Double layer —	146
2.4	Interface between electrode and electrolyte —	148
2.5	Faradaic reactions —	148
3	Porous electrodes —	149
3.1	Ideally polarizable porous matrix —	149
3.2	Species transport —	150
3.3	Darcy flow —	151
3.4	Further effects —	151
3.5	Coupling between porous electrodes and free flow —	152
4	Numerical approximation by Voronoi finite volumes —	153
4.1	Description of the method —	154
4.2	Flux expressions for scalar convection diffusion —	156
4.3	Coupling to flow problems —	157
4.4	Software —	158
5	Conclusions —	159

Benjamin Ganis, Ruijie Liu, Bin Wang, Mary F. Wheeler, and Ivan Yotov

Multiscale modeling of flow and geomechanics — 165

1	Introduction —	165
2	Background —	167
2.1	Multidomain methodology —	167
2.2	Discretization methods —	169
2.3	Coupled flow and geomechanics —	172
3	Multiscale multiphysics discretizations for flow and geomechanics —	175
4	Multiscale domain decomposition solvers and preconditioners —	177
5	A posteriori error estimation and time-stepping —	181
6	Uncertainty quantification, verification, and validation —	183
7	Applications —	186
7.1	Compositional modeling of multiphase flow —	186
7.2	Fixed stress iterative coupling scheme —	188
7.3	Plasticity modeling —	191
8	Summary and conclusions —	195

List of contributors — 207

Index — 209