

Contents

Preface	vii
Preface to the second edition	ix
1 Introduction to tropical geometry	1
1.1 Images under the logarithm	1
1.2 Families of amoebas	4
1.3 Non-Archimedean amoebas	5
1.4 Non-standard complex numbers	7
1.5 The tropical semifield $\mathbb{T}$	9
1.6 Tropical curves and integer affine structure	11
1.7 Exercises	14
2 Patchworking of algebraic varieties	17
2.1 Introduction: A general idea of the patchworking construction . . .	17
2.2 Elements of toric geometry	19
2.2.1 Construction of toric varieties	19
2.2.2 A toric variety associated with a fan	20
2.2.3 A toric variety associated with a convex lattice polyhedron	21
2.2.4 Embedding of $\text{Tor}(\Delta)$ into a projective space	22
2.2.5 The real part of a toric variety and the moment map	22
2.2.6 Hypersurfaces in toric varieties	24
2.3 Viro's patchworking method	25
2.3.1 Chart of a real polynomial	25
2.3.2 Patchworking of real nonsingular hypersurfaces	27
2.3.3 Combinatorial patchworking	30
2.3.4 A tropical point of view on the combinatorial Viro patch- working	34
2.3.5 Patchworking of pseudo-holomorphic curves on ruled surfaces	37
2.4 Patchworking of singular algebraic hypersurfaces	45
2.4.1 Initial data	46
2.4.2 Transversality conditions	46

2.4.3	The patchworking theorem	47
2.4.4	Some $\mathcal{S}$ -transversality criteria	50
2.5	Tropicalization and patchworking in the enumeration of nodal curves	51
2.5.1	Plane tropical curves	52
2.5.2	Algebraic enumerative problem and its tropical analogue . .	54
2.5.3	Tropical formulas for the Gromov–Witten and Welschinger invariants	56
2.5.4	Tropical limit	56
2.5.5	Tropicalization of nodal curves	57
2.5.6	Reconstruction of a simple tropical curve	61
2.5.7	Reconstruction of the limit curve $C^{(0)}$	63
2.5.8	Refinement of a tropical limit	64
2.5.9	Refinement of the condition to pass through a fixed point .	67
2.5.10	Refined patchworking theorem	69
2.5.11	The real case: Welschinger invariants	73
2.6	Exercises	74
3	Applications of tropical geometry to enumerative geometry	77
3.1	Introduction	77
3.2	Tropical hypersurfaces in $\mathbb{R}^n$	78
3.3	Geometric description of plane tropical curves	81
3.4	Count of complex nodal curves	83
3.5	Correspondence theorem	84
3.6	Mikhalkin’s algorithm	85
3.7	Welschinger invariants	86
3.8	Welschinger invariants W_m for small m	88
3.9	Tropical calculation of Welschinger invariants	89
3.10	Asymptotic enumeration of real rational curves	90
3.11	Recurrence formula for Welschinger invariants	92
3.12	Welschinger invariants $W_{m,i}$	93
3.13	Exercises	95
	List of Figures	97
	Bibliography	99