## **Contents**

Pre	face		vii		
1	An introduction using classical examples				
	1.1	Numerical differentiation. First look at the problem of regularization.  The balancing principle	1 1 3		
		1.1.3 A posteriori choice of the stepsize	6 9 10		
	1.2	<ul><li>1.1.5 The balancing principle in a general framework</li></ul>			
		1.2.1 Summation methods          1.2.2 Deterministic noise model	12 13 14 15		
		<ul> <li>1.2.4 Smoothness associated with a basis</li></ul>	18 19 21		
	1.3	1.3.1 Natural linearization of the elliptic Cauchy problem  1.3.2 Regularization by discretization	25 27 36 39		
2	Basics of single parameter regularization schemes				
	2.1	Simple example for motivation	47		
	2.2	Essentially ill-posed linear operator equations. Least-squares solution.  General view on regularization			
	2.3	Smoothness in the context of the problem. Benchmark accuracy levels for deterministic and stochastic data noise models	65		
			68		



		2.3.2	noise model	73
	2.4	-	al order and the saturation of regularization methods in Hilbert	80
	2.5	_	ing the penalty term for variance reduction. Regularization in t scales	90
	2.6	Estima	tion of linear functionals from indirect noisy observations	101
	2.7	7 Regularization by finite-dimensional approximation		
	2.8	Model 2.8.1 2.8.2 2.8.3	selection based on indirect observation in Gaussian white noise Linear models given by least-squares methods Operator monotone functions The problem of model selection (continuation)	127 131
	2.9		ning example: an operator equation formulation is not always ate (numerical differentiation revisited)	
		2.9.2 2.9.3	Error bounds in $L_2$	147
		2.9.4	Numerical differentiation in the space of continuous functions	
		2.9.5	Relation to the Savitzky–Golay method. Numerical examples	
3	Mul	ltiparan	•	163
-	3.1	-	do we really need multiparameter regularization?	
	3.2		arameter discrepancy principle	
		3.2.2	principle	
		3.2.3	parameters satisfying the discrepancy principle  Properties of the model function approximation	172
		3.2.4	Discrepancy curve and the convergence analysis	173
		3.2.5	Heuristic algorithm for the model function approximation of the multiparameter discrepancy principle	174
		3.2.6	Generalization in the case of more than two regularization parameters	175
	3.3	Numer	ical realization and testing	177
		3.3.1	Numerical examples and comparison	177

Contents xiii

		3.3.2	Two-parameter discrepancy curve		
		3.3.3	A numerical check of Proposition 3.1 and use of a discrepancy		
		3.3.4	curve Experiments with three-parameter regularization		
	3.4		arameter regularization with one negative parameter for		
		_	ms with noisy operators and right-hand side		
		3.4.1	Computational aspects for regularized total least squares		
		3.4.2	Computational aspects for dual regularized total least squares.		
		3.4.3 3.4.4	Error bounds in the case $B = I$		
		3.4.5	Numerical illustrations. Model function approximation in dual	193	
		3.7.3	regularized total least squares	197	
4	Reg	ulariza	tion algorithms in learning theory	203	
	4.1		rised learning problem as an operator equation in a reproducing		
		kernel	Hilbert space (RKHS)		
		4.1.1 4.1.2	Reproducing kernel Hilbert spaces and related operators A priori assumption on the problem: general source	205	
			conditions	207	
	4.2	Kernel	independent learning rates	209	
		4.2.1	Regularization for binary classification: risk bounds and		
			Bayes consistency	217	
	4.3	-	ve kernel methods using the balancing principle		
		4.3.1	Adaptive learning when the error measure is known		
		4.3.2	Adaptive learning when the error measure is unknown		
		4.3.3 4.3.4	Proofs of Propositions 4.6 and 4.7		
	4.4			231	
	4.4		adaptive regularization with application to blood glucose	225	
		4.4.1	-	233	
		1. 1.1	current measurements	242	
	4.5	Multip	parameter regularization in learning theory	249	
5			ing approach to regularization – case study: blood glucose		
		diction	v v	255	
	5.1	A brief	f introduction to meta-learning and blood glucose prediction	255	
	5.2	A tradi	itional learning theory approach: issues and concerns	259	
	5.3	Meta-learning approach to choosing a kernel and a regularization			
		_	eter		
		5.3.1	Optimization operation	263	

	٠	
X	1	v

		5.3.2	Heuristic operation	267
		5.3.3	Learning at metalevel	267
	5.4	Case-study: blood glucose prediction		269
Bibliography				
Index				