

Contents

1	Introduction	1
1.1	Epistemology of the Cosmos	1
1.2	Areas of Research Presented in this Thesis	3
	References	5
2	Cosmology Background	7
2.1	Prelude: An Historical Overview	7
2.2	The Friedmann Lemaître Robertson Walker Universe	10
2.3	Cosmological Distance Measures	15
2.3.1	Conformal Time	15
2.3.2	Redshift and Comoving Distance	16
2.3.3	Luminosity Distance	18
2.3.4	Angular Diameter Distance	20
2.4	Inflation	20
2.5	The Evolution of the Inhomogeneities	23
2.5.1	Photon and Baryon Perturbations	25
2.5.2	Cold Dark Matter Perturbations	27
2.6	CMB Angular Power Spectrum	27
2.7	Matter Power Spectrum and Baryon Acoustic Oscillations	32
	References	34
3	Dark Energy and Apparent Late Time Acceleration	37
3.1	Dark Energy Models	38
3.1.1	The Cosmological Constant and the Energy Density of the Vacuum	38
3.1.2	Λ CDM and More General Dark Energy Models	41
3.2	A Note on Modified Gravity	42
3.3	Alternatives: Voids and the Backreaction	43
	References	44

4	Supernovae Ia	45
4.1	Spectral Classification of SNe Ia	45
4.2	Progenitor Scenarios for SNe Ia	47
4.3	Detonation and Deflagration Models	48
4.4	SNe Ia as Standardizable Candles	49
4.5	Light Curve Fitting	50
4.5.1	SALT-II Light Curve Fitting	51
4.5.2	MLCS2k2 Light Curve Fitting	53
	References	54
5	Statistical Techniques	57
5.1	Elements of Bayesian Statistics	57
5.1.1	Bayesian Parameter Inference	58
5.1.2	Priors and the Bayesian Definition of a Model	59
5.1.3	Bayesian Model Selection	60
5.2	Numerical Methods	62
5.2.1	Markov Chain Monte Carlo Techniques	64
5.2.2	Nested Sampling	67
5.3	Forecasting: Fisher Matrix Formalism	70
5.4	Some Aspects of Frequentist Statistics	71
5.4.1	Maximum Likelihood and the Chi Square Statistic	72
	References	73
6	Bayesian Doubt: Should We Doubt the Cosmological Constant?	75
6.1	Introduction	75
6.2	Bayesian Doubt and Model Discovery	76
6.2.1	Bayesian Doubt Described	76
6.2.2	Bayesian Evidence of the Unknown Model	78
6.2.3	Behaviour of Doubt and the Posterior Probability for Λ CDM	80
6.3	Application of Doubt to the Dark Energy Equation of State	81
6.3.1	Parametrization of the Unknown Model	83
6.3.2	Numerical Implementation and Data Sets	84
6.4	Results and Discussion	85
6.5	Impact of the Addition of Further Known Models	89
6.6	Conclusions of Bayesian Doubt	92
	References	92
7	Bayesian Parameter Inference for SNe Ia Data	95
7.1	Motivations for This Work	95
7.2	Notation and a Description of the Problem	95
7.3	The Standard χ^2 Method	97

7.4	In Search of a Bayesian Solution	99
7.4.1	Identifying the Bias Problem.	99
7.4.2	Locating the Cause of the Bias Problem.	102
7.4.3	Bias Solved in the Linear Toy Model	106
7.4.4	Gull's Estimator for a	113
7.4.5	Comparison Between Gull's Estimator for a and the Marginalized Simple χ^2 Expression	114
7.5	Bayesian Hierarchical Model (BHM): Description	117
7.6	Bayesian Hierarchical Model: Calculation.	119
7.6.1	A Note on Notation	119
7.6.2	Calculation Expressed in Matrix Notation.	120
7.6.3	Integration Over Intrinsic Redshifts	124
7.6.4	Integration Over Latent $\{c_\star, x_\star, M\}$	125
7.6.5	Integration Over Population Variables $\{c_\star, x_\star, M_0\}$	127
7.7	Numerical Trials with Simulated Data	128
7.7.1	Description of the Real SNe Ia Data Sets	128
7.7.2	Description of the Simulated SNe Ia Data Sets	129
7.7.3	Numerical Sampling.	131
7.7.4	Parameter Reconstruction	133
7.7.5	Comparison of Performance of the Two Methods Over 100 Trials	134
7.8	Cosmological Constraints from Current SNIa Data Using BHM.	140
7.9	Bayesian Hierarchical Model Future Planned Work	142
7.9.1	Applications Using SNe Ia BHM.	142
7.9.2	Bayesian Hierarchical Network Analysis of Gamma Ray-Burst Data	145
7.10	Conclusions.	146
	References	147

8 Robustness to Systematic Error for Future

	Dark Energy Probes	149
8.1	Figures of Merit for Future Dark Energy Probes	150
8.1.1	Gaussian Linear Model.	150
8.1.2	The Statistical Figure of Merit	151
8.1.3	Robustness of Dark Energy Probes	152
8.1.4	The Robustness Figure of Merit	154
8.2	Generalization to Arbitrary Number of Probes.	157
8.3	Properties of the Robustness FoM	158
8.4	Robustness of Future Dark Energy Probes	163
8.4.1	Future SN Ia Measurements	163
8.4.2	Future Baryonic Acoustic Oscillations Measurements.	165

8.5	Results	169
8.6	Conclusions.	171
	References	171
9	Summary and Conclusions	173
	Index	175