

# Contents

## Part I Prologue

|          |                                                      |          |
|----------|------------------------------------------------------|----------|
| <b>1</b> | <b>Recollections from Elementary Quantum Physics</b> | <b>3</b> |
|          | References                                           | 8        |

## Part II Two-State Quantum Systems

|          |                                       |           |
|----------|---------------------------------------|-----------|
| <b>2</b> | <b>A Most Simple Two-Level System</b> | <b>11</b> |
| 2.1      | Magnetic Moment and Spin              | 11        |
| 2.2      | Zeeman Effect                         | 13        |
| 2.3      | Stern–Gerlach Effect                  | 15        |
|          | References                            | 16        |
| <b>3</b> | <b>Quantum Theory in a Nutshell</b>   | <b>17</b> |
| 3.1      | Spin Matrices                         | 17        |
| 3.2      | Energy Matrices                       | 19        |
| 3.3      | Expectation Values                    | 21        |
| 3.3.1    | Expectation Value of Energy           | 22        |
| 3.3.2    | Expectation Value of Spin             | 23        |
| 3.4      | Uncertainties                         | 25        |
| 3.4.1    | Uncertainty of Spin                   | 26        |
| 3.4.2    | Uncertainty of Energy                 | 27        |
| 3.5      | Spin Precession                       | 28        |
| 3.5.1    | Longitudinal Field                    | 28        |
| 3.5.2    | Transverse Field                      | 29        |
|          | References                            | 32        |
| <b>4</b> | <b>Experiments on Spin Precession</b> | <b>33</b> |
| 4.1      | Muon Spin Precession                  | 33        |
| 4.2      | Light Scattering                      | 35        |
| 4.3      | Spinor Rotation Through $720^\circ$   | 38        |
|          | References                            | 40        |

|           |                                                             |     |
|-----------|-------------------------------------------------------------|-----|
| <b>5</b>  | <b>General Solution for the Two-Level System</b>            | 41  |
| 5.1       | Matrix Diagonalization                                      | 42  |
| 5.2       | Construction of the Eigenvectors                            | 43  |
| 5.3       | The Time Dependent Solution                                 | 45  |
| 5.3.1     | Evolution of an Energy Eigenstate                           | 45  |
| 5.3.2     | Evolution of an Angular-Momentum Eigenstate                 | 46  |
| <b>6</b>  | <b>Other Tools and Concepts</b>                             | 49  |
| 6.1       | Time Evolution Operator                                     | 49  |
| 6.2       | Rotation Matrices                                           | 50  |
| 6.3       | Projection Operators                                        | 51  |
| 6.4       | Pure States and Mixed States                                | 53  |
| 6.5       | The Density Matrix                                          | 56  |
| 6.6       | Coherence and Interference                                  | 59  |
| 6.7       | Dirac's Bra-Ket Notation                                    | 62  |
| <b>7</b>  | <b>Diabolic Points, Geometric Phases, and Quantum Chaos</b> | 65  |
| 7.1       | Level Crossings and Level Repulsions                        | 65  |
| 7.1.1     | The Field Dependence of Energy and of Polarization          | 65  |
| 7.1.2     | Level Repulsion in a Spin- $\frac{1}{2}$ Systems            | 67  |
| 7.1.3     | Level Repulsion in a Spin-1 System                          | 68  |
| 7.2       | The Adiabatic Theorem                                       | 70  |
| 7.3       | Geometric Phases                                            | 72  |
| 7.3.1     | Derivation of the Berry Phase                               | 72  |
| 7.3.2     | Excursions in Magnetic-Field Space                          | 74  |
| 7.3.3     | Excursions in the Space of Shapes                           | 77  |
| 7.3.4     | The Aharonov–Bohm Effect                                    | 79  |
| 7.4       | Quantum Chaos                                               | 80  |
|           | References                                                  | 84  |
| <b>8</b>  | <b>The Coupling of Particles</b>                            | 85  |
| 8.1       | Bosons and Fermions                                         | 85  |
| 8.2       | The Coupling of Spins                                       | 87  |
| 8.3       | Example: Hyperfine Structure                                | 90  |
| <b>9</b>  | <b>“Spooky Action at a Distance”</b>                        | 93  |
| 9.1       | Quantum Entanglement                                        | 93  |
| 9.2       | Bell's Inequalities                                         | 97  |
|           | References                                                  | 100 |
| <b>10</b> | <b>The Heisenberg Equation of Motion</b>                    | 101 |
| 10.1      | Matrix Mechanics                                            | 101 |
| 10.2      | Commutation Relations and Uncertainty Principle             | 105 |
| 10.3      | The Bloch Equations                                         | 107 |
|           | References                                                  | 110 |

Part III Quantum Physics at Work

11 Spin Resonance . . . . . 113

11.1 Basics of Spin Resonance . . . . . 114

11.2 Methods of Spin Resonance . . . . . 116

11.3 Applications of Spin Resonance . . . . . 121

References . . . . . 124

12 Two-State Systems in Atomic and Molecular Physics . . . . . 125

12.1 Photons as Two-State Systems . . . . . 125

12.2 Optical Resonance Transitions . . . . . 126

12.3 Optical Analogies of Spin Rotation and Spin Resonance . . . . . 128

12.4 Particles in a Double Well . . . . . 131

12.4.1 The NH<sub>3</sub> Molecule . . . . . 131

12.4.2 The Ammonia Maser . . . . . 133

12.4.3 Bose–Einstein Condensate in a Double Trap . . . . . 134

References . . . . . 136

13 Two-State Systems in Condensed Matter . . . . . 137

13.1 Glasses . . . . . 137

13.2 Josephson Effects . . . . . 139

13.2.1 Basics of Superconductivity . . . . . 139

13.2.2 Josephson Junctions and Their Applications . . . . . 141

References . . . . . 144

14 Two-State Systems in Nuclear and Particle Physics . . . . . 147

14.1 Isospin . . . . . 147

14.2 Flavor and Color . . . . . 151

14.3 Particle Oscillations . . . . . 153

14.3.1 Kaon Oscillations . . . . . 153

14.3.2 Neutrino Oscillations . . . . . 155

14.3.3 Neutron Oscillations . . . . . 156

References . . . . . 158

15 Quantum Informatics . . . . . 159

15.1 Quantum Information Theory . . . . . 159

15.2 Quantum Computing and Quantum Communication . . . . . 161

References . . . . . 163

Part IV Multilevel Systems and Tensor Operators

16 Rotations and Angular Momentum . . . . . 167

16.1 Symmetries . . . . . 167

16.2 Properties of Angular Momentum . . . . . 170

16.3 Representations . . . . . 172

16.4 The Spherical Harmonics . . . . . 173

16.5 The Rotation Matrices . . . . . 175

|           |                                                       |     |
|-----------|-------------------------------------------------------|-----|
| <b>17</b> | <b>Irreducible Tensors</b>                            | 179 |
| 17.1      | Scalars, Vectors, and Tensors                         | 179 |
| 17.2      | Properties of Irreducible Tensors                     | 180 |
| 17.2.1    | Definition of Irreducible Tensors                     | 180 |
| 17.2.2    | A More Practical Definition                           | 182 |
| 17.2.3    | Simple Examples                                       | 183 |
| 17.3      | The Coupling of Irreducible Tensors                   | 184 |
| 17.3.1    | The Coupling of Angular Momenta                       | 185 |
| 17.3.2    | General Tensor Coupling                               | 187 |
| 17.3.3    | Some Special Cases                                    | 188 |
| 17.4      | The Wigner–Eckart Theorem                             | 190 |
| <b>18</b> | <b>Electromagnetic Multipole Interactions</b>         | 193 |
| 18.1      | Static Magnetic Interactions                          | 193 |
| 18.2      | Static Electric Interactions                          | 194 |
| 18.2.1    | Multipole Expansion of Electrostatic Energy           | 194 |
| 18.2.2    | Electric Quadrupole Interaction                       | 195 |
| 18.3      | Selection Rules for Electromagnetic Transitions       | 198 |
|           | References                                            | 202 |
| <b>19</b> | <b>The Generalized Spin Precession Equation</b>       | 203 |
| 19.1      | The Density Matrix                                    | 203 |
| 19.1.1    | Definition of the Density Matrix                      | 203 |
| 19.1.2    | The Liouville Equation of Motion                      | 205 |
| 19.2      | Some Preparative Steps                                | 207 |
| 19.2.1    | Normalized Irreducible Tensor Operators               | 207 |
| 19.2.2    | A Bra-Ket Notation for Tensor Operators               | 208 |
| 19.3      | The Irreducible Components of the Density Matrix      | 211 |
| 19.3.1    | Definition of the Statistical Tensors                 | 211 |
| 19.3.2    | Simple Examples of Statistical Tensors                | 212 |
| 19.4      | The Liouville Equation for the Statistical Tensors    | 214 |
|           | References                                            | 215 |
| <b>20</b> | <b>Reorientation in Static Electromagnetic Fields</b> | 217 |
| 20.1      | Magnetic Dipole Precession                            | 217 |
| 20.2      | Electric Quadrupole Reorientation                     | 218 |
| 20.3      | Reorientation in Mixed Magnetic and Electric Fields   | 219 |
| 20.4      | Time Average Results                                  | 220 |
| 20.5      | Angular Distribution of Radiation                     | 222 |
| 20.5.1    | Asymmetric $\beta$ -Decay                             | 222 |
| 20.5.2    | Anisotropic Photon Emission                           | 224 |
|           | References                                            | 226 |
| <b>21</b> | <b>Reorientation in Time Dependent Fields</b>         | 227 |
| 21.1      | Radiofrequency Irradiation in a Magnetic Field        | 227 |
| 21.1.1    | Density Operator in the Rotating Frame                | 227 |
| 21.1.2    | Rotating Wave Approximation                           | 229 |

|           |                                                                                     |            |
|-----------|-------------------------------------------------------------------------------------|------------|
| 21.1.3    | Statistical Tensors in the Rotating Wave Approximation . . .                        | 229        |
| 21.1.4    | Time Average Results . . . . .                                                      | 230        |
| 21.2      | Multiple Quantum Transitions . . . . .                                              | 231        |
| 21.3      | Dressed Atoms . . . . .                                                             | 233        |
| 21.3.1    | Dressed Atoms and the Floquet Theorem . . . . .                                     | 233        |
| 21.3.2    | Dressed Atoms and Second Quantization . . . . .                                     | 235        |
| 21.3.3    | A Dressed Neutron Experiment . . . . .                                              | 236        |
| 21.3.4    | Outlook on Nonclassical Photon Interactions . . . . .                               | 237        |
|           | References . . . . .                                                                | 238        |
| <b>22</b> | <b>Relaxation and Decoherence . . . . .</b>                                         | <b>239</b> |
| 22.1      | General Features . . . . .                                                          | 239        |
| 22.2      | The Perturbative Approach . . . . .                                                 | 241        |
| 22.3      | The Stochastic Approach . . . . .                                                   | 243        |
| 22.4      | Decoherence . . . . .                                                               | 248        |
|           | References . . . . .                                                                | 253        |
|           | <b>Appendices . . . . .</b>                                                         | <b>255</b> |
| A.1       | The Orthogonality of the Irreducible Tensor Operators . . . . .                     | 255        |
| A.2       | The Reduced Matrix Element $\langle j    \hat{T}_L(j)    j \rangle$ . . . . .       | 256        |
| A.3       | The Reduced Matrix Element $\langle L    \hat{T}_{L_2}(j)    L_1 \rangle$ . . . . . | 257        |
| A.4       | Spherical Harmonics . . . . .                                                       | 258        |
| A.5       | Rotation Matrices . . . . .                                                         | 259        |
| A.6       | Clebsch–Gordan Coefficients . . . . .                                               | 260        |
| A.7       | Normalized Irreducible Tensor Operators . . . . .                                   | 261        |
| A.8       | Coefficients of the Generalized Precession Equation . . . . .                       | 261        |
| A.9       | Transforming away Part of an Interaction from the Liouville<br>Equation . . . . .   | 262        |
|           | <b>Index . . . . .</b>                                                              | <b>263</b> |