Contents

Part I Lock-Based Synchronization

1	The I	Mutual E	Exclusion Problem	3
	1.1	Multip	rocess Program	3
		1.1.1	The Concept of a Sequential Process	3
		1.1.2	The Concept of a Multiprocess Program	4
	1.2	Proces	s Synchronization	4
		1.2.1	Processors and Processes	4
		1.2.2	Synchronization	4
		1.2.3	Synchronization: Competition	5
		1.2.4	Synchronization: Cooperation	7
		1.2.5	The Aim of Synchronization	
			Is to Preserve Invariants	7
	1.3	The M	Iutual Exclusion Problem	9
		1.3.1	The Mutual Exclusion Problem (Mutex)	9
		1.3.2	Lock Object	11
•		1.3.3	Three Families of Solutions	12
	1.4	Summ	ary	13
	1.5		graphic Notes	13
2	Solvi	ng Mutu	al Exclusion	15
	2.1		Based on Atomic Read/Write Registers	15
		2.1.1	Atomic Register	15
		2.1.2	Mutex for Two Processes:	
			An Incremental Construction	17
		2.1.3	A Two-Process Algorithm	19
		2.1.4	Mutex for <i>n</i> Processes:	
			Generalizing the Previous Two-Process Algorithm	22
		2.1.5	Mutex for <i>n</i> Processes:	
			A Tournament-Based Algorithm	26
		2.1.6	A Concurrency-Abortable Algorithm	29
			-	

xii Contents

		2.1.7	A Fast Mutex Algorithm	33
		2.1.8	Mutual Exclusion in a Synchronous System	37
	2.2	Mutex	Based on Specialized Hardware Primitives	38
		2.2.1	Test&Set, Swap and Compare&Swap	39
		2.2.2	From Deadlock-Freedom to Starvation-Freedom	40
		2.2.3		44
	2.3	Mutex	Without Atomicity	45
		2.3.1	•	45
		2.3.2	The Bakery Mutex Algorithm	48
		2.3.3		53
	2.4		ary	58
	2.5			58
	2.6		2 1	59
3	Lock-	Based C	Concurrent Objects	61
-	3.1			61
		3.1.1		61
		3.1.2		62
	3.2	A Base		63
		3.2.1	, i	63
		3.2.2	Using Semaphores to Solve	
				65
		3.2.3	Using Semaphores to Solve	
				71
		3.2.4	Using Semaphores to Solve	
			the Readers-Writers Problem	74
		3.2.5	Using a Buffer to Reduce Delays	
			· ·	78
	3.3	A Con	struct for Imperative Languages: the Monitor	8
		3.3.1		82
		3.3.2		83
		3.3.3		85
		3.3.4		8
		3.3.5	Monitors for the Readers-Writers Problem	89
		3.3.6	Scheduled Wait Operation	94
	3.4	Declar	rative Synchronization: Path Expressions	95
		3.4.1	Definition	96
		3.4.2	Using Path Expressions to Solve	
			Synchronization Problems	97
		3.4.3	A Semaphore-Based Implementation	
	•		of Path Expressions	98
	3.5			0
	3.6			.02
	3.7	Exerci	ses and Problems	.02

Contents xiii

Part II O	n the	Foundations	Side: The	e Atomicit	y Concept
-----------	-------	--------------------	-----------	------------	-----------

4	Aton	nicity: Formal Definition and Properties	113
	4.1	Introduction	113
	4.2	Computation Model	115
		4.2.1 Processes and Operations	115
		4.2.2 Objects	116
		4.2.3 Histories	117
		4.2.4 Sequential History	119
	4.3	Atomicity	120
		4.3.1 Legal History	120
		4.3.2 The Case of Complete Histories	121
		4.3.3 The Case of Partial Histories	123
	4.4	Object Composability and Guaranteed	
		Termination Property	125
		4.4.1 Atomic Objects Compose for Free	125
		4.4.2 Guaranteed Termination	127
	4.5	Alternatives to Atomicity	128
		4.5.1 Sequential Consistency	128
		4.5.2 Serializability	130
	4.6	Summary	131
			
	4.7	Bibliographic Notes	132
Pa		•	132
Pa 5	4.7	Bibliographic Notes	132
	4.7	Bibliographic Notes	
	4.7 rt III Mute	Bibliographic Notes	135
	4.7 rt III Mute	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion	135 135
	4.7 rt III Mute	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions. 5.1.1 The Mutex-Freedom Notion.	135 135 135
	4.7 rt III Mute	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions	135 135 135 137
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from	135 135 135 137
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers.	135 135 135 137
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from	135 135 137 140 140
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers.	135 135 137 140 140
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers. 5.2.2 A Simple Obstruction-Free Object from	135 135 135 137 140 140
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions. 5.1.1 The Mutex-Freedom Notion. 5.1.2 Progress Conditions. 5.1.3 Non-blocking with Respect to Wait-Freedom. Mutex-Free Concurrent Objects. 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers. 5.2.2 A Simple Obstruction-Free Object from Read/Write Registers. 5.2.3 A Remark on Compare&Swap: The ABA Problem. 5.2.4 A Non-blocking Queue Based on	135 135 137 140 140 143 143
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions. 5.1.1 The Mutex-Freedom Notion. 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom. Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers. 5.2.2 A Simple Obstruction-Free Object from Read/Write Registers. 5.2.3 A Remark on Compare&Swap: The ABA Problem. 5.2.4 A Non-blocking Queue Based on Read/Write Registers and Compare&Swap.	135 135 135 137 140 140
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers. 5.2.2 A Simple Obstruction-Free Object from Read/Write Registers. 5.2.3 A Remark on Compare&Swap: The ABA Problem. 5.2.4 A Non-blocking Queue Based on Read/Write Registers and Compare&Swap 5.2.5 A Non-blocking Stack Based on	135 135 135 137 140 140 143 145
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers. 5.2.2 A Simple Obstruction-Free Object from Read/Write Registers 5.2.3 A Remark on Compare&Swap: The ABA Problem. 5.2.4 A Non-blocking Queue Based on Read/Write Registers and Compare&Swap 5.2.5 A Non-blocking Stack Based on Compare&Swap Registers	135 135 137 140 140 143 143
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers. 5.2.2 A Simple Obstruction-Free Object from Read/Write Registers. 5.2.3 A Remark on Compare&Swap: The ABA Problem. 5.2.4 A Non-blocking Queue Based on Read/Write Registers and Compare&Swap 5.2.5 A Non-blocking Stack Based on Compare&Swap Registers 5.2.6 A Wait-Free Stack Based on	135 135 135 137 140 140 143 145 146
	4.7 rt III Muta 5.1	Mutex-Free Synchronization ex-Free Concurrent Objects Mutex-Freedom and Progress Conditions 5.1.1 The Mutex-Freedom Notion 5.1.2 Progress Conditions 5.1.3 Non-blocking with Respect to Wait-Freedom Mutex-Free Concurrent Objects 5.2.1 The Splitter: A Simple Wait-Free Object from Read/Write Registers. 5.2.2 A Simple Obstruction-Free Object from Read/Write Registers 5.2.3 A Remark on Compare&Swap: The ABA Problem. 5.2.4 A Non-blocking Queue Based on Read/Write Registers and Compare&Swap 5.2.5 A Non-blocking Stack Based on Compare&Swap Registers	135 135 135 137 140 140 143 145

xiv Contents

	5.3		ng Obstruction-Freedom to Stronger Progress	
			Read/Write Model	155
		5.3.1	Failure Detectors	155
		5.3.2	Contention Managers for Obstruction-Free	
			Object Implementations	157
		5.3.3	Boosting Obstruction-Freedom to Non-blocking	158
		5.3.4	Boosting Obstruction-Freedom to Wait-Freedom	159
		5.3.5	Mutex-Freedom Versus Loops Inside a Contention	
			Manager Operation	161
	5.4	Summa	ary	162
	5.5	Bibliog	graphic Notes	162
	5.6	Exercis	ses and Problems	163
6	Hybri	id Concu	ırrent Objects	165
	6.1	The No	otion of a Hybrid Implementation	165
		6.1.1	Lock-Based Versus Mutex-Free Operation:	
			Static Hybrid Implementation	166
		6.1.2	Contention Sensitive (or Dynamic Hybrid)	
			Implementation	166
		6.1.3	The Case of Process Crashes	166
	6.2		ic Hybrid Implementation of a Concurrent Set Object	167
	•	6.2.1	Definition and Assumptions	167
		6.2.2	Internal Representation and Operation	
			Implementation	167
		6.2.3	Properties of the Implementation	171
	6.3		ntion-Sensitive Implementations	172
	0.0	6.3.1	Contention-Sensitive Binary Consensus	172
		6.3.2	A Contention Sensitive Non-blocking	
		3.5	Double-Ended Queue	176
	6.4	The N	otion of an Abortable Object	181
		6.4.1	Concurrency-Abortable Object	181
		6.4.2	From a Non-blocking Abortable Object	
			to a Starvation-Free Object	183
	6.5	Summ	ary	186
	6.6		graphic Notes	186
	6.7		ses and Problems	187
7	Wait-	-Free Ob	ojects from Read/Write Registers Only	189
	7.1		it-Free Weak Counter for Infinitely Many Processes	189
		7.1.1	A Simple Counter Object	190
	•	7.1.2	Weak Counter Object for Infinitely Many Processes	191
		7.1.3	A One-Shot Weak Counter Wait-Free Algorithm	193
		7.1.4	Proof of the One-Shot Implementation	194
		7.1.5	A Multi-Shot Weak Counter Wait-Free Algorithm	199

Contents

	7.2	Store-Collect Object	201
		7.2.1 Store-Collect Object: Definition	201
		7.2.2 An Adaptive Store-Collect Implementation	204
		7.2.3 Proof and Cost of the Adaptive Implementation	208
	7.3	Fast Store-Collect Object	211
	, ,,	7.3.1 Fast Store-Collect Object: Definition	211
		7.3.2 A Fast Algorithm for the store_collect() Operation	212
		7.3.3 Proof of the Fast Store-Collect Algorithm	215
	7.4	Summary	217
	7.5	Bibliographic Notes	217
	7.6	Problem	218
8	Snone	hot Objects from Read/Write Registers Only	219
o	8.1	Snapshot Objects: Definition	219
	8.2	Single-Writer Snapshot Object	220
	0.2	8.2.1 An Obstruction-Free Implementation	221
		8.2.2 From Obstruction-Freedom to Bounded	221
		Wait-Freedom	223
		8.2.3 One-Shot Single-Writer Snapshot Object:	
		Containment Property	227
	8.3	Single-Writer Snapshot Object with Infinitely	22,
	8.3	Many Processes	228
	8.4	Multi-Writer Snapshot Object.	230
	0.4	8.4.1 The Strong Freshness Property	231
		8.4.2 An Implementation of a Multi-Writer	231
		Snapshot Object	231
		8.4.3 Proof of the Implementation	234
	8.5	Immediate Snapshot Objects	238
	8.5	8.5.1 One-Shot Immediate Snapshot Object: Definition	238
		8.5.2 One-Shot Immediate Snapshot Versus	230
		One-Shot Snapshot	238
			230
		8.5.3 An Implementation of One-Shot	240
		Immediate Snapshot Objects	2 4 0
		8.5.4 A Recursive Implementation of a One-Shot Immediate Snapshot Object	244
	0.6		247
	8.6	Summary	247
	8.7	Bibliographic Notes	248
	8.8	Problem	248

xvi Contents

9	Renar	ning Ob	jects from Read/Write Registers Only	249
	9.1		ing Objects	249
		9.1.1	The Base Renaming Problem	249
		9.1.2	One-Shot Renaming Object	250
		9.1.3	Adaptive Implementations	250
		9.1.4	A Fundamental Result	251
		9.1.5	Long-Lived Renaming	252
	9.2	Non-tri	iviality of the Renaming Problem	252
	9.3	A Split	tter-Based Optimal Time-Adaptive Implementation	254
	9.4	A Snap	oshot-Based Optimal Size-Adaptive Implementation	256
		9.4.1	A Snapshot-Based Implementation	256
		9.4.2	Proof of the Implementation	258
	9.5	Recurs	ive Store-Collect-Based Size-Adaptive	
		Implen	nentation	259
		9.5.1	A Recursive Renaming Algorithm	259
		9.5.2	An Example	262
		9.5.3	Proof of the Renaming Implementation	263
	9.6	Varian	t of the Previous Recursion-Based	
		Renam	ning Algorithm	266
		9.6.1	A Renaming Implementation Based	
			on Immediate Snapshot Objects	266
		9.6.2	An Example of a Renaming Execution	268
	9.7	Long-I	Lived Perfect Renaming Based	
		on Tes	st&Set Registers	269
		9.7.1	Perfect Adaptive Renaming	269
		9.7.2	Perfect Long-Lived Test&Set-Based Renaming	270
	9.8	Summa	ary	271
	9.9	Bibliog	graphic Notes	271
	9.10	Exerci	ses and Problems	272
Par	t IV	The Tra	nsactional Memory Approach	
	-			277
10			Memory	277
	10.1		Are Software Transactional Memories	277
		10.1.1	Transactions = High-Level Synchronization	277
		10.1.2	At the Programming Level	279
	10.2		System	281
		10.2.1	7	201
		1000	and Abort of a Transaction	281
		10.2.2	1 1	282
		10.2.3		282
		10.2.4	Incremental Reads and Deferred Updates	283

Contents xvii

		10.2.5	Read-Only Versus Update Transactions	283
		10.2.6	Read Invisibility	284
	10.3	A Logi	cal Clock-Based STM System: TL2	284
		10.3.1	Underlying System and Control Variables	
			of the STM System	284
		10.3.2	Underlying Principle: Consistency	
			with Respect to Transaction Birth Date	285
		10.3.3	The Implementation of an Update Transaction	286
		10.3.4	The Implementation of a Read-Only Transaction	288
	10.4		ion-Based STM System: JVSTM	289
	10.4	10.4.1	Underlying and Control Variables	
		10.7.1	of the STM System	290
		10.4.2	The Implementation of an Update Transaction	291
		10.4.2	The Implementation of a Read-Only Transaction	293
	10.5		or Clock-Based STM System	293
	10.5			293
		10.5.1	The Virtual World Consistency Condition	295
		10.5.2	An STM System for Virtual World Consistency	293
		10.5.3	The Algorithms Implementing	207
	10.6		the STM Operations	296
	10.6		ary	299
	10.7	_	graphic Notes	299
	10.8	Exercis	ses and Problems	300
_			1.4. (1.1)	
Par			oundations Side:	
	F	rom Sat	e Bits to Atomic Registers	
11	Sofo 1	Dogulon	and Atomic Read/Write Registers	305
11			Regular, and Atomic Registers	305
	11.1			305
		11.1.1	Reminder: The Many Faces of a Register	308
		11.1.2	From Regularity to Atomicity: A Theorem	300
		11.1.3	A Fundamental Problem:	310
	110	T 1/	The Construction of Registers	
	11.2		ery Simple Bounded Constructions	311
		11.2.1	Safe/Regular Registers:	211
			From Single-Reader to Multi-Reader	311
		11.2.2	Binary Multi-Reader Registers:	210
			From Safe to Regular	313
	11.3		Bits to b-Valued Registers	314
		11.3.1	From Safe Bits to b-Valued Safe Registers	314
		11.3.2	From Regular Bits to Regular b-Valued Registers	315
		11.3.3	From Atomic Bits to Atomic b-Valued Registers	319

xviii Contents

	11.4		Inbounded Constructions	321
		11.4.1	SWSR Registers:	
			From Unbounded Regular to Atomic	322
		11.4.2	Atomic Registers:	
			From Unbounded SWSR to SWMR	324
		11.4.3	Atomic Registers:	
			From Unbounded SWMR to MWMR	325
	11.5		ry	327
	11.6	Bibliogr	raphic Notes	327
12	From	Safe Bits	s to Atomic Bits:	
-			and Optimal Construction	329
	12.1		er Bound Theorem	329
		12.1.1	Two Preliminary Lemmas	330
		12.1.2	The Lower Bound Theorem	331
	12.2		truction of an Atomic Bit from Three Safe Bits	334
		12.2.1	Base Architecture of the Construction	334
		12.2.2	Underlying Principle and Signaling Scheme	335
		12.2.3	The Algorithm Implementing	
			the Operation R.write()	33€
		12.2.4	The Algorithm Implementing	
			the Operation R.read()	336
		12.2.5	Cost of the Construction	338
	12.3	Proof of	f the Construction of an Atomic Bit	338
		12.3.1	A Preliminary Theorem	338
		12.3.2	Proof of the Construction	340
	12.4	Summai	ry	344
	12.5	Bibliog	raphic Notes	345
	12.6	Exercise	e	345
13	Dound	ad Const	tructions of Atomic b-Valued Registers	347
13	13.1		ction	347
	13.1		sion-Free (Pure Buffers) Construction	349
	13.2	13.2.1	Internal Representation of the Atomic b-Valued	547
		13.2.1	Register R	349
		13.2.2	Underlying Principle: Two-Level Switch	
			to Ensure Collision-Free Accesses to Buffers	349
		13.2.3	The Algorithms Implementing	
	•		the Operations R .write() and R .read()	350
		13.2.4	Proof of the Construction: Collision-Freedom	352
		13.2.5	Correctness Proof	355
	13.3	A Cons	truction Based on Impure Buffers	357
		13.3.1	Internal Representation of the Atomic b-Valued	
			Register <i>R</i>	357

Contents xix

		13.3.2	An Incremental Construction	358
		13.3.3	The Algorithms Implementing	260
			the Operations R.write() and R.read()	360
		13.3.4 13.3.5	Proof of the Construction	360
			Atomic Register	367
	13.4	Summa	ry	368
	13.5	Bibliog	raphic Notes	368
Par	t VI	On the F	oundations Side:	
			putability Power of Concurrent Objects (Consensus))
14	Unive	rsality of	Consensus	371
	14.1	Univers	sal Object, Universal Construction,	
		and Co	nsensus Object	371
		14.1.1	Universal (Synchronization) Object	
			and Universal Construction	371
		14.1.2	The Notion of a Consensus Object	372
	14.2	Inputs a	and Base Principles of Universal Constructions	373
		14.2.1	The Specification of the Constructed Object	373
		14.2.2	Base Principles of Universal Constructions	374
	14.3	An Unb	bounded Wait-Free Universal Construction	374
		14.3.1	Principles and Description of the Construction	375
		14.3.2	Proof of the Construction	378
		14.3.3	Non-deterministic Objects	382
		14.3.4	Wait-Freedom Versus Bounded Wait-Freedom	383
	14.4	A Bour	nded Wait-Free Universal Construction	384
		14.4.1	Principles of the Construction	384
		14.4.2	Proof of the Construction	388
		14.4.3	Non-deterministic Objects	391
	14.5	From E	Binary Consensus to Multi-Valued Consensus	391
		14.5.1	A Construction Based on the Bit Representation	
			of Proposed Values	392
		14.5.2	A Construction for Unbounded Proposed Values	394
	14.6	Summa	ıry	395
	14.7	Bibliog	raphic Notes	396
	14.8	Exercis	ses and Problems	396
15	The C	Case of U	nreliable Base Objects	399
	15.1	Respon	sive Versus Non-responsive Crash Failures	400
	15.2	SWSR	Registers Prone to Crash Failures	400
		15.2.1	Reliable Register When Crash Failures	
			Are Responsive: An Unbounded Construction	401

xx Contents

		15.2.2	Reliable Register When Crash Failures Are				
			Responsive: A Bounded Construction	403			
		15.2.3	Reliable Register When Crash Failures Are Not				
			Responsive: An Unbounded Construction	406			
	15.3	Consen	sus When Crash Failures Are Responsive:				
		A Bour	nded Construction	408			
		15.3.1	The "Parallel Invocation" Approach				
			Does Not Work	408			
		15.3.2	A t-Tolerant Wait-Free Construction	409			
		15.3.3	Consensus When Crash Failures Are Not Responsive:				
			An Impossibility	410			
	15.4	Omissi	on and Arbitrary Failures	410			
		15.4.1	Object Failure Modes	410			
		15.4.2	Simple Examples	412			
		15.4.3	Graceful Degradation	413			
		15.4.4	Fault-Tolerance Versus Graceful Degradation	417			
	15.5	Summa	ıry	418			
	15.6	Bibliog	raphic Notes	419			
	15.7	Exercis	ses and Problems	419			
16			mbers and the Consensus Hierarchy	421			
	16.1		onsensus Number Notion	421			
	16.2		nentals	422			
		16.2.1	Schedule, Configuration, and Valence	422			
		16.2.2	Bivalent Initial Configuration	423			
	16.3		eak Wait-Free Power of Atomic Registers	425			
		16.3.1	The Consensus Number of Atomic				
			Read/Write Registers Is 1	425			
		16.3.2	The Wait-Free Limit of Atomic Registers	428			
	16.4		s Whose Consensus Number Is 2	429			
		16.4.1	Consensus from Test&Set Objects	429			
		16.4.2	Consensus from Queue Objects	431			
		16.4.3	Consensus from Swap Objects	432			
		16.4.4	Other Objects for Wait-Free Consensus				
			in a System of Two Processes	432			
		16.4.5	Power and Limit of the Previous Objects	433			
	16.5	-	s Whose Consensus Number Is $+\infty$	438			
		16.5.1	Consensus from Compare&Swap Objects	439			
		16.5.2	Consensus from Mem-to-Mem-Swap Objects	440			
		16.5.3	Consensus from an Augmented Queue	442			
		16.5.4	From a Sticky Bit to Binary Consensus	442			
		16.5.5	Impossibility Result	443			

Contents xxi

	16.6	Hierarchy of Atomic Objects	443
		16.6.1 From Consensus Numbers to a Hierarchy	443
		16.6.2 On Fault Masking	444
		16.6.3 Robustness of the Hierarchy	445
	16.7	Summary	445
	16.8	Bibliographic Notes	445
	16.9	Exercises and Problems	446
17		lpha(s) and Omega of Consensus:	
	Failur	e Detector-Based Consensus	449
	17.1	De-constructing Compare&Swap	450
	17.2	A Liveness-Oriented Abstraction: The Failure Detector Ω	452
		17.2.1 Definition of Ω	452
		17.2.2 Ω-Based Consensus:	
		Ω as a Resource Allocator or a Scheduler	453
	17.3	Three Safety-Oriented Abstractions:	
		Alpha ₁ , Alpha ₂ , and Alpha ₃	454
		17.3.1 A Round-Free Abstraction: Alpha ₁	454
		17.3.2 A Round-Based Abstraction: Alpha ₂	455
		17.3.3 Another Round-Free Abstraction: Alpha ₃	456
		17.3.4 The Rounds Seen as a Resource	457
	17.4	Ω -Based Consensus	457
		17.4.1 Consensus from Alpha ₁ Objects and Ω	457
		17.4.2 Consensus from an Alpha ₂ Object and Ω	459
		17.4.3 Consensus from an Alpha ₃ Object and Ω	460
		17.4.4 When the Eventual Leader Elected by Ω	
		Does Not Participate	463
		17.4.5 The Notion of an Indulgent Algorithm	464
		17.4.6 Consensus Object Versus Ω	464
	17.5	Wait-Free Implementations of the Alpha ₁ and Alpha ₂	
		Abstractions	465
		17.5.1 Alpha ₁ from Atomic Registers	465
		17.5.2 Alpha ₂ from Regular Registers	467
	17.6	Wait-Free Implementations of the Alpha ₂ Abstraction	
		from Shared Disks	472
		17.6.1 Alpha ₂ from Unreliable Read/Write Disks	472
		17.6.2 Alpha ₂ from Active Disks	476
	17.7	Implementing Ω	477
		17.7.1 The Additional Timing Assumption <i>EWB</i>	478
		17.7.2 An <i>EWB</i> -Based Implementation of Ω	479
		17.7.3 Proof of the Construction	481
		17.7.4 Discussion	484

xxii			Contents
	17.9	Summary	. 485
Afte	rword .		. 489
Bibl	iograph	ny	. 495