

# Contents

<b>Chapter 1</b>	<b>Modular forms and the Shimura-Taniyama Conjecture</b>	<b>1</b>
1.1	Elliptic functions	1
1.2	Modular forms	3
1.3	Examples	5
1.4	Hecke operators and eigenforms	8
1.5	$L$ -functions	10
1.6	Modular forms of higher level	12
1.7	Elliptic curves	13
1.8	Conjectures, and the theorem of Wiles, et al.	15
	References	16
<b>Chapter 2</b>	<b>Periods of automorphic forms</b>	<b>17</b>
2.1	Automorphic forms	17
2.2	Periods arising from Dirichlet series	19
2.3	Periods arising from elliptic curves	20
2.4	Periods arising from cohomology theory	22
2.5	Recapitulation	23
2.6	Hilbert modular forms	23
2.7	Automorphic forms with respect to a quaternion algebra	25
2.8	Adelic automorphic forms	27
2.9	Hecke operators	29
2.10	The standard $L$ -function for Hilbert modular forms and the $V$ periods	32
2.11	The inner product and the $Q$ periods	35
2.12	The Eichler-Shimura Isomorphism and the $P$ periods	36
	References	42
<b>Chapter 3</b>	<b>Lifting of automorphic forms</b>	<b>43</b>
3.1	Dirichlet series	43
3.2	Hecke characters with respect to a real quadratic number	

field .....	44
3.3 $L$ -series and the base change lift for characters .....	46
3.4 Hilbert modular forms over a real quadratic field and the Doi-Naganuma lift .....	48
3.5 Applications of base change .....	51
3.6 Shimura's period conjectures .....	52
3.7 Yoshida's work .....	54
References .....	55
<b>Chapter 4 Zeros of <math>L</math>-functions</b> .....	57
4.1 Trivial zeros .....	57
4.2 The Grand Riemann Hypothesis .....	60
4.3 Zero-free regions .....	62
4.4 Siegel zeros .....	67
4.5 Correlations of zeros .....	71
4.6 Random matrix theory .....	73
References .....	75
<b>Chapter 5 Special values of <math>L</math>-functions</b> .....	77
5.1 The Birch-Swinnerton-Dyer conjecture .....	77
5.2 The Gross-Zagier formula .....	80
5.3 The class number problem .....	82
5.4 The Lindelöf conjecture and subconvexity bounds .....	85
5.5 Discrete moment conjecture of $L$ -functions .....	89
5.6 Integral moment conjectures of $L$ -functions .....	94
References .....	97
<b>Chapter 6 Theta lifts and periods with respect to a quadratic extension</b> .....	99
6.1 Setting .....	99
6.2 Automorphic forms with respect to $B_E$ .....	100
6.3 Theta function .....	103
6.4 Transformation formulas .....	106
6.5 Theta correspondence .....	107
6.6 Reduction of the integral .....	107
6.7 Two technical lemmata .....	109
6.8 Main theorems on theta correspondence .....	110

6.9	Adelic forms on $\mathcal{G}'_{\mathbb{A}}$ .....	110
6.10	Embeddings and inner product .....	111
6.11	Theta correspondence of adelic forms .....	113
6.12	Hecke Operators on $GL_2(F)$ .....	115
6.13	The action of Hecke operators on $I(z; C; r; h)$ .....	117
6.14	Hecke operators on adelic forms .....	118
6.15	Relation between actions of Hecke operators .....	120
6.16	Further Reading .....	121
	References .....	122