

Contents

*When we have not what we like,
we must like what we have.*
COMTE DE BUSSY-RABUTIN

1	Basic Graph Theory	1
1.1	Graphs, Subgraphs and Factors	2
1.2	Paths, Cycles, Connectedness, Trees	5
1.3	Euler Tours	13
1.4	Hamiltonian Cycles	15
1.5	Planar Graphs	21
1.6	Digraphs	25
1.7	An Application: Tournaments and Leagues	29
2	Algorithms and Complexity	35
2.1	Algorithms	36
2.2	Representing Graphs	38
2.3	The Algorithm of Hierholzer	41
2.4	How to Write Down Algorithms	44
2.5	The Complexity of Algorithms	46
2.6	Directed Acyclic Graphs	49
2.7	An Introduction to NP-completeness	52
2.8	Five NP-complete Problems	56
3	Shortest Paths	65
3.1	Shortest Paths	65
3.2	Finite Metric Spaces	68
3.3	Breadth First Search and Bipartite Graphs	69
3.4	Shortest Path Trees	74
3.5	Bellman's Equations and Acyclic Networks	76
3.6	An Application: Scheduling Projects	79
3.7	The Algorithm of Dijkstra	83
3.8	An Application: Train Schedules	88
3.9	The Algorithm of Floyd and Warshall	91
3.10	Cycles of Negative Length	96
3.11	Path Algebras	97

4	Spanning Trees	103
4.1	Trees and Forests	103
4.2	Incidence Matrices	105
4.3	Minimal Spanning Trees	110
4.4	The Algorithms of Prim, Kruskal and Boruvka	113
4.5	Maximal Spanning Trees	120
4.6	Steiner Trees	122
4.7	Spanning Trees with Restrictions	125
4.8	Arborescences and Directed Euler Tours	130
5	The Greedy Algorithm	135
5.1	The Greedy Algorithm and Matroids	135
5.2	Characterizations of Matroids	137
5.3	Matroid Duality	143
5.4	The Greedy Algorithm as an Approximation Method	146
5.5	Minimization in Independence Systems	153
5.6	Accessible Set Systems	157
6	Flows	163
6.1	The Theorems of Ford and Fulkerson	163
6.2	The Algorithm of Edmonds and Karp	170
6.3	Auxiliary Networks and Phases	177
6.4	Constructing Blocking Flows	186
6.5	Zero-One Flows	194
6.6	The Algorithm of Goldberg and Tarjan	198
6.7	Further Reading	214
7	Combinatorial Applications	219
7.1	Disjoint Paths: Menger's Theorem	219
7.2	Matchings: König's Theorem	224
7.3	Partial Transversals: The Marriage Theorem	228
7.4	Combinatorics of Matrices	234
7.5	Dissections: Dilworth's Theorem	239
7.6	Parallelisms: Baranyai's Theorem	243
7.7	Supply and Demand: The Gale-Ryser Theorem	246
8	Connectivity and Depth First Search	251
8.1	k -connected Graphs	251
8.2	Depth First Search	254
8.3	2-connected Graphs	258
8.4	Depth First Search for Digraphs	264
8.5	Strongly Connected Digraphs	266
8.6	Edge Connectivity	270
9	Colorings	275
9.1	Vertex Colorings	275
9.2	Comparability Graphs and Interval Graphs	279

9.3	Edge Colorings	282
9.4	Cayley Graphs	285
9.5	The Five Color Theorem	290
10	Circulations	295
10.1	Circulations and Flows	295
10.2	Feasible Circulations	298
10.3	Elementary Circulations	305
10.4	The Algorithm of Klein	312
10.5	The Algorithm of Busacker and Gowen	315
10.6	Potentials and ϵ -optimality	319
10.7	Optimal Circulations by Successive Approximation	327
10.8	A Polynomial Procedure REFINE	332
10.9	The Minimum Mean Cycle Cancelling Algorithm	339
10.10	Some Further Problems	344
10.11	An Application: Graphical Codes	347
11	The Network Simplex Algorithm	359
11.1	The Minimum Cost Flow Problem	360
11.2	Tree Solutions	362
11.3	Constructing an Admissible Tree Structure	365
11.4	The Algorithm	370
11.5	Efficient Implementations	375
12	Synthesis of Networks	379
12.1	Symmetric Networks	379
12.2	Synthesis of Equivalent Flow Trees	382
12.3	Synthesizing Minimal Networks	389
12.4	Cut Trees	395
12.5	Increasing the Capacities	400
13	Matchings	405
13.1	The Berge-Tutte Formula	405
13.2	Augmenting Paths	408
13.3	Alternating Trees and Blossoms	412
13.4	The Algorithm of Edmonds	418
13.5	The Gallai-Edmonds Structure Theorem	434
13.6	Matching Matroids	438
14	Weighted Matchings	441
14.1	The Bipartite Case	442
14.2	The Hungarian Algorithm	443
14.3	Matchings, Linear Programs, and Polytopes	452
14.4	The General Case	457
14.5	The Chinese Postman	461
14.6	Matchings and Shortest Paths	466
14.7	Some Further Problems	473
14.8	An Application: Decoding Graphical Codes	475

15 A Hard Problem: The TSP 481

15.1 Basic Definitions 481

15.2 Lower Bounds: Relaxations 484

15.3 Lower Bounds: Subgradient Optimization 490

15.4 Approximation Algorithms 495

15.5 Upper Bounds: Heuristics 502

15.6 Upper Bounds: Local Search 504

15.7 Exact Neighborhoods and Suboptimality 509

15.8 Optimal Solutions: Branch and Bound 515

15.9 Concluding Remarks 523

Appendix A Some NP-Complete Problems 527

Appendix B Solutions 537

B.1 Solutions for Chap. 1 537

B.2 Solutions for Chap. 2 543

B.3 Solutions for Chap. 3 548

B.4 Solutions for Chap. 4 555

B.5 Solutions for Chap. 5 561

B.6 Solutions for Chap. 6 565

B.7 Solutions for Chap. 7 574

B.8 Solutions for Chap. 8 583

B.9 Solutions for Chap. 9 589

B.10 Solutions for Chap. 10 592

B.11 Solutions for Chap. 11 601

B.12 Solutions for Chap. 12 602

B.13 Solutions for Chap. 13 606

B.14 Solutions for Chap. 14 613

B.15 Solutions for Chap. 15 619

Appendix C List of Symbols 623

C.1 General Symbols 623

C.2 Special Symbols 624

References 629

Index 661