

Contents

1	Infinitesimal Calculus, Consistently and Accessibly	1
1.1	An Accessible Axiom System for Infinitesimal Calculus: Minimal Internal Set Theory	1
1.2	Finer Classification of the Reals: Finite vs. Limited	4
2	Radically Elementary Probability Theory	7
2.1	Random Variables and Stochastic Processes.....	7
2.2	Integrability and Limitedness.....	9
2.3	Wiener Walks and Wiener Processes.....	13
2.4	Distribution of the Wiener Walk.....	15
2.5	Integrability and Limited Paths of the Wiener Walk	16
3	Radically Elementary Stochastic Integrals	19
3.1	Martingales and Itô Integrals	19
3.2	Radically Elementary Itô Processes	23
3.3	A Basic Radically Elementary Itô Formula	26
3.4	Analytic Excursion: A Radically Elementary Treatment of Geometric Itô Processes with Monotone Drift	28
3.5	The Radically Elementary Version of Lévy's Characterization of Wiener Processes.....	33
4	The Radically Elementary Girsanov Theorem and the Diffusion Invariance Principle.....	35
4.1	Girsanov's Theorem	35
4.2	The Radically Elementary Diffusion Invariance Principle	42
5	Excursion to Financial Economics: A Radically Elementary Approach to the Fundamental Theorems of Asset Pricing	45
6	Excursion to Financial Engineering: Volatility Invariance in the Black–Scholes Model	55

7	A Radically Elementary Theory of Itô Diffusions and Associated Partial Differential Equations	61
7.1	Itô Diffusions	61
7.2	The Markov Property of Itô Diffusions and the Feynman–Kac Formula	64
8	Excursion to Mathematical Physics: A Radically Elementary Definition of Feynman Path Integrals	71
9	A Radically Elementary Theory of Lévy Processes	77
9.1	Random Walks and Lévy Walks	77
9.2	Integrability of Lévy Walks with Limited Increments	80
9.3	Lindstrøm’s Characterization of Lévy Walks	82
9.4	A Radically Elementary Itô–Doebelin Formula for Lévy Walks with Limited-Variation Jump Part	86
9.5	A Brief Look at Lévy Finance	90
10	Final Remarks	93
A	Excursion to Logic: Some Remarks on the Metamathematics of Minimal Internal Set Theory	95
A.1	An Alternative Road to Minimal Internal Set Theory	95
A.2	A Simple Relative Consistency Proof for a Substantial Subsystem of minIST	97
A.3	Definable Models for (Minimal) Nonstandard Analysis	101
B	Robinsonian vs. Minimal Nonstandard Analysis	103
	References	107
	Index	111