

TABLE OF CONTENTS

I	<u>History of algebraic curves</u>	1
1.	Origin and generation of curves	2
1.1	The circle and the straight line	2
1.2	The classical problems of antiquity	3
1.3	The conic sections	4
1.4	The cissoid of Diocles	9
1.5	The conchoid of Nicomedes	13
1.6	The spiric sections of Perseus	16
1.7	From the epicycles of Hipparchos to the Wankel motor	19
1.8	Caustics and contour curves in optics and perspective	41
1.9	Further examples of curves from science and technology	58
2.	Synthetic and analytic geometry	66
2.1	Coordinates	66
2.2	The development of analytic geometry	69
2.3	Equations for curves	71
2.4	Examples of the application of analytic methods	79
2.5	Newton's investigation of cubic curves	87
3.	The development of projective geometry	102
3.1	Descriptive geometry and projective geometry	102
3.2	The development of analytic projective geometry	106
3.3	The projective plane as a manifold	118
3.4	Complex projective geometry	136
II	<u>Investigation of curves by elementary algebraic methods</u>	172
4.	Polynomials	172
4.1	Decomposition into prime factors	173
4.2	Divisibility properties of polynomials	174
4.3	Zeroes of polynomials	190
4.4	Homogeneous and inhomogeneous polynomials	193

5.	Definition and elementary properties of plane algebraic curves	202
5.1	Decomposition into irreducible components	202
5.2	Intersection of a curve by a line	208
5.3	Singular points of plane curves	211
6.	The intersection of plane curves	227
6.1	Bézout's theorem	227
6.2	Applications of Bézout's theorem	239
6.3	The intersection ring of $P_2(\mathbb{C})$	260
7.	Some simple types of curves	278
7.1	Quadrics	278
7.2	Linear systems of cubics	281
7.3	Inflection point figures and normal forms of cubics	283
7.4	Cubics, elliptic curves and abelian varieties	306
III	<u>Investigation of curves by resolution of singularities</u>	323
8.	Local investigations	325
8.1	Localisation-local rings	325
8.2	Singularities as analytic set germs	337
8.3	Newton polygons and Puiseux expansions	370
8.4	Resolution of singularities by quadratic transformations	455
8.5	Topology of singularities	535
9.	Global investigations	576
9.1	The Plücker formulae	576
9.2	The formulae of Clebsch and Noether	601
9.3	Differential forms on Riemann surfaces and their periods	627
	Bibliography	694
	Index	702