

Contents

Preface	v
I Basic structures	
1 Vectors and operators	3
1.1 Hilbert space	3
1.2 Operators	4
1.3 Positivity	5
1.4 Trace and duality	6
1.5 Convexity	8
1.6 Notes and references	9
2 States, observables, statistics	10
2.1 Structure of statistical theories	10
2.1.1 Classical systems	10
2.1.2 Axioms of statistical description	11
2.2 Quantum states	14
2.3 Quantum observables	16
2.3.1 Quantum observables from the axioms	16
2.3.2 Compatibility and complementarity	18
2.3.3 The uncertainty relation	21
2.3.4 Convex structure of observables	22
2.4 Statistical discrimination between quantum states	25
2.4.1 Formulation of the problem	25
2.4.2 Optimal observables	25
2.5 Notes and references	31
3 Composite systems and entanglement	34
3.1 Composite systems	34
3.1.1 Tensor products	34
3.1.2 Naimark's dilation	36
3.1.3 Schmidt decomposition and purification	38

3.2	Quantum entanglement vs “local realism”	41
3.2.1	Paradox of Einstein–Podolski–Rosen and Bell’s inequalities	41
3.2.2	Mermin–Peres game	45
3.3	Quantum systems as information carriers	47
3.3.1	Transmission of classical information	47
3.3.2	Entanglement and local operations	48
3.3.3	Superdense coding	49
3.3.4	Quantum teleportation	50
3.4	Notes and references	52

II The primary coding theorems

4	Classical entropy and information	57
4.1	Entropy of a random variable and data compression	57
4.2	Conditional entropy and the Shannon information	59
4.3	The Shannon capacity of the classical noisy channel	62
4.4	The channel coding theorem	64
4.5	Wiretap channel	69
4.6	Gaussian channel	71
4.7	Notes and references	72
5	The classical-quantum channel	74
5.1	Codes and achievable rates	74
5.2	Formulation of the coding theorem	75
5.3	The upper bound	78
5.4	Proof of the weak converse	83
5.5	Typical projectors	87
5.6	Proof of the Direct Coding Theorem	92
5.7	The reliability function for pure-state channel	95
5.8	Notes and references	98

III Channels and entropies

6	Quantum evolutions and channels	103
6.1	Quantum evolutions	103
6.2	Completely positive maps	106
6.3	Definition of the channel	112

6.4	Entanglement-breaking and PPT channels	114
6.5	Quantum measurement processes	117
6.6	Complementary channels	119
6.7	Covariant channels	124
6.8	Qubit channels	127
6.9	Notes and references	129
7	Quantum entropy and information quantities	132
7.1	Quantum relative entropy	132
7.2	Monotonicity of the relative entropy	133
7.3	Strong subadditivity of the quantum entropy	138
7.4	Continuity properties	140
7.5	Information correlation, entanglement of formation and conditional entropy	142
7.6	Entropy exchange	147
7.7	Quantum mutual information	149
7.8	Notes and references	151
IV	Basic channel capacities	
8	The classical capacity of quantum channel	155
8.1	The coding theorem	155
8.2	The χ -capacity	157
8.3	The additivity problem	160
8.3.1	The effect of entanglement in encoding and decoding	160
8.3.2	A hierarchy of additivity properties	164
8.3.3	Some entropy inequalities	166
8.3.4	Additivity for complementary channels	169
8.3.5	Nonadditivity of quantum entropy quantities	171
8.4	Notes and references	178
9	Entanglement-assisted classical communication	180
9.1	The gain of entanglement assistance	180
9.2	The classical capacities of quantum observables	184
9.3	Proof of the Converse Coding Theorem	188
9.4	Proof of the Direct Coding Theorem	190
9.5	Notes and references	194

10	Transmission of quantum information	195
10.1	Quantum error-correcting codes	195
10.1.1	Error correction by repetition	195
10.1.2	General formulation	197
10.1.3	Necessary and sufficient conditions for error correction	198
10.1.4	Coherent information and perfect error correction	200
10.2	Fidelities for quantum information	203
10.2.1	Fidelities for pure states	203
10.2.2	Relations between the fidelity measures	205
10.2.3	Fidelity and the Bures distance	208
10.3	The quantum capacity	210
10.3.1	Achievable rates	210
10.3.2	The quantum capacity and the coherent information	215
10.3.3	Degradable channels	217
10.4	The private classical capacity and the quantum capacity	220
10.4.1	The quantum wiretap channel	220
10.4.2	Proof of the Private Capacity Theorem	223
10.4.3	Large deviations for random operators	229
10.4.4	The Direct Coding Theorem for the quantum capacity	232
10.5	Notes and references	237
V	Infinite systems	
11	Channels with constrained inputs	243
11.1	Convergence of density operators	243
11.2	Quantum entropy and relative entropy	247
11.3	Constrained c-q channel	249
11.4	Classical-quantum channel with continuous alphabet	252
11.5	Constrained quantum channel	254
11.6	Entanglement-assisted capacity of constrained channels	257
11.7	Entanglement-breaking channels in infinite dimensions	259
11.8	Notes and references	264
12	Gaussian systems	266
12.1	Preliminary material	266
12.1.1	Spectral decomposition and Stone's Theorem	266
12.1.2	Operators associated with the Heisenberg commutation relation	269

12.1.3	Classical signal plus quantum noise	272
12.1.4	The classical-quantum Gaussian channel	275
12.2	Canonical commutation relations	276
12.2.1	Weyl–Segal CCR	276
12.2.2	The symplectic space	279
12.2.3	Dynamics, quadratic operators and gauge transformations	281
12.3	Gaussian states	284
12.3.1	Characteristic function	284
12.3.2	Definition and properties of Gaussian states	285
12.3.3	The density operator of Gaussian state	289
12.3.4	Entropy of a Gaussian state	290
12.3.5	Separability and purification	293
12.4	Gaussian channels	296
12.4.1	Open bosonic systems	296
12.4.2	Gaussian channels: basic properties	300
12.4.3	Gaussian observables	301
12.4.4	Gaussian entanglement-breaking channels	303
12.5	The capacities of Gaussian channels	307
12.5.1	Maximization of the mutual information	307
12.5.2	Gauge-covariant channels	308
12.5.3	Maximization of the coherent information	310
12.5.4	The classical capacity: conjectures	311
12.6	The case of one mode	314
12.6.1	Classification of Gaussian channels	314
12.6.2	Entanglement-breaking channels	320
12.6.3	Attenuation/amplification/classical noise channel	321
12.6.4	Estimating the quantum capacity	325
12.7	Notes and references	329
	Bibliography	333
	Index	346