

Contents

Part I The Basic Equations

1	Coordinates, Mass Distribution, and Gravitational Field in Spherical Stars	3
1.1	Eulerian Description	3
1.2	Lagrangian Description	4
1.3	The Gravitational Field	6
2	Conservation of Momentum	9
2.1	Hydrostatic Equilibrium	9
2.2	The Role of Density and Simple Solutions	10
2.3	Simple Estimates of Central Values P_c, T_c	12
2.4	The Equation of Motion for Spherical Symmetry	13
2.5	The Non-spherical Case	15
2.6	Hydrostatic Equilibrium in General Relativity	15
2.7	The Piston Model	17
3	The Virial Theorem	19
3.1	Stars in Hydrostatic Equilibrium	19
3.2	The Virial Theorem of the Piston Model	21
3.3	The Kelvin–Helmholtz Timescale	22
3.4	The Virial Theorem for Non-vanishing Surface Pressure	23
4	Conservation of Energy	25
4.1	Thermodynamic Relations	25
4.2	The Perfect Gas and the Mean Molecular Weight	28
4.3	Thermodynamic Quantities for the Perfect, Monatomic Gas	30
4.4	Energy Conservation in Stars	31
4.5	Global and Local Energy Conservation	33
4.6	Timescales	35

5	Transport of Energy by Radiation and Conduction	37
5.1	Radiative Transport of Energy	37
5.1.1	Basic Estimates	37
5.1.2	Diffusion of Radiative Energy	38
5.1.3	The Rosseland Mean for κ_v	40
5.2	Conductive Transport of Energy	42
5.3	The Thermal Adjustment Time of a Star	43
5.4	Thermal Properties of the Piston Model	45
6	Stability Against Local, Non-spherical Perturbations	47
6.1	Dynamical Instability	47
6.2	Oscillation of a Displaced Element	52
6.3	Vibrational Stability	54
6.4	The Thermal Adjustment Time	55
6.5	Secular Instability	56
6.6	The Stability of the Piston Model	58
7	Transport of Energy by Convection	61
7.1	The Basic Picture	62
7.2	Dimensionless Equations	65
7.3	Limiting Cases, Solutions, Discussion	66
7.4	Extensions of the Mixing-Length Theory	70
8	The Chemical Composition	73
8.1	Relative Mass Abundances	73
8.2	Variation of Composition with Time	74
8.2.1	Radiative Regions	74
8.2.2	Diffusion	76
8.2.3	Convective Regions	80
9	Mass Loss	83

Part II The Overall Problem

10	The Differential Equations of Stellar Evolution	89
10.1	The Full Set of Equations	89
10.2	Timescales and Simplifications	91
11	Boundary Conditions	93
11.1	Central Conditions	93
11.2	Surface Conditions	95
11.3	Influence of the Surface Conditions and Properties of Envelope Solutions	98
11.3.1	Radiative Envelopes	98
11.3.2	Convective Envelopes	101
11.3.3	Summary	102
11.3.4	The $T-r$ Stratification	102

12 Numerical Procedure	105
12.1 The Shooting Method	105
12.2 The Henyey Method	106
12.3 Treatment of the First- and Second-Order Time Derivatives	113
12.4 Treatment of the Diffusion Equation	115
12.5 Treatment of Mass Loss	117
12.6 Existence and Uniqueness	118

Part III Properties of Stellar Matter

13 The Perfect Gas with Radiation	123
13.1 Radiation Pressure	123
13.2 Thermodynamic Quantities	124
14 Ionization	127
14.1 The Boltzmann and Saha Formulae	127
14.2 Ionization of Hydrogen	130
14.3 Thermodynamical Quantities for a Pure Hydrogen Gas	132
14.4 Hydrogen–Helium Mixtures	133
14.5 The General Case	135
14.6 Limitation of the Saha Formula	137
15 The Degenerate Electron Gas	139
15.1 Consequences of the Pauli Principle	139
15.2 The Completely Degenerate Electron Gas	140
15.3 Limiting Cases	144
15.4 Partial Degeneracy of the Electron Gas	145
16 The Equation of State of Stellar Matter	151
16.1 The Ion Gas	151
16.2 The Equation of State	152
16.3 Thermodynamic Quantities	154
16.4 Crystallization	157
16.5 Neutronization	158
16.6 Real Gas Effects	159
17 Opacity	163
17.1 Electron Scattering	163
17.2 Absorption Due to Free–Free Transitions	164
17.3 Bound–Free Transitions	165
17.4 Bound–Bound Transitions	166
17.5 The Negative Hydrogen Ion	168
17.6 Conduction	169
17.7 Molecular Opacities	170
17.8 Opacity Tables	172

18	Nuclear Energy Production	175
18.1	Basic Considerations	175
18.2	Nuclear Cross Sections	179
18.3	Thermonuclear Reaction Rates	182
18.4	Electron Shielding	188
18.5	The Major Nuclear Burning Stages	192
18.5.1	Hydrogen Burning	193
18.5.2	Helium Burning	197
18.5.3	Carbon Burning and Beyond	199
18.6	Neutron-Capture Nucleosynthesis	201
18.7	Neutrinos	205

Part IV Simple Stellar Models

19	Polytropic Gaseous Spheres	213
19.1	Polytropic Relations	213
19.2	Polytropic Stellar Models	215
19.3	Properties of the Solutions	216
19.4	Application to Stars	218
19.5	Radiation Pressure and the Polytrope $n = 3$	219
19.6	Polytropic Stellar Models with Fixed K	220
19.7	Chandrasekhar's Limiting Mass	221
19.8	Isothermal Spheres of an Ideal Gas	222
19.9	Gravitational and Total Energy for Polytropes	224
19.10	Supermassive Stars	226
19.11	A Collapsing Polytrope	227
20	Homology Relations	233
20.1	Definitions and Basic Relations	233
20.2	Applications to Simple Material Functions	237
20.2.1	The Case $\delta = 0$	237
20.2.2	The Case $\alpha = \delta = \varphi = 1, a = b = 0$	237
20.2.3	The Role of the Equation of State	239
20.3	Homologous Contraction	241
21	Simple Models in the U-V Plane	243
21.1	The U - V Plane	243
21.2	Radiative Envelope Solutions	246
21.3	Fitting of a Convective Core	248
21.4	Fitting of an Isothermal Core	250
22	The Zero-Age Main Sequence	251
22.1	Surface Values	251
22.2	Interior Solutions	254
22.3	Convective Regions	258
22.4	Extreme Values of M	260
22.5	The Eddington Luminosity	261

23 Other Main Sequences	263
23.1 The Helium Main Sequence	263
23.2 The Carbon Main Sequence	266
23.3 Generalized Main Sequences	267
24 The Hayashi Line	271
24.1 Luminosity of Fully Convective Models	272
24.2 A Simple Description of the Hayashi Line	273
24.3 The Neighbourhood of the Hayashi Line and the Forbidden Region	276
24.4 Numerical Results	279
24.5 Limitations for Fully Convective Models	281
25 Stability Considerations	283
25.1 General Remarks	283
25.2 Stability of the Piston Model	285
25.2.1 Dynamical Stability	285
25.2.2 Inclusion of Non-adiabatic Effects	286
25.3 Stellar Stability	288
25.3.1 Perturbation Equations	289
25.3.2 Dynamical Stability	290
25.3.3 Non-adiabatic Effects	292
25.3.4 The Gravothermal Specific Heat	293
25.3.5 Secular Stability Behaviour of Nuclear Burning	294

Part V Early Stellar Evolution

26 The Onset of Star Formation	299
26.1 The Jeans Criterion	299
26.1.1 An Infinite Homogeneous Medium	299
26.1.2 A Plane-Parallel Layer in Hydrostatic Equilibrium	302
26.2 Instability in the Spherical Case	303
26.3 Fragmentation	307
27 The Formation of Protostars	311
27.1 Free-Fall Collapse of a Homogeneous Sphere	311
27.2 Collapse onto a Condensed Object	313
27.3 A Collapse Calculation	314
27.4 The Optically Thin Phase and the Formation of a Hydrostatic Core	315
27.5 Core Collapse	317
27.6 Evolution in the Hertzsprung–Russell Diagram	320
28 Pre-Main-Sequence Contraction	323
28.1 Homologous Contraction of a Gaseous Sphere	323
28.2 Approach to the Zero-Age Main Sequence	326

29	From the Initial to the Present Sun	329
29.1	Known Solar Data	329
29.2	Choosing the Initial Model	331
29.3	A Standard Solar Model	333
29.4	Results of Helioseismology	336
29.5	Solar Neutrinos	338
30	Evolution on the Main Sequence	343
30.1	Change in the Hydrogen Content	343
30.2	Evolution in the Hertzsprung–Russell Diagram	346
30.3	Timescales for Central Hydrogen Burning	347
30.4	Complications Connected with Convection	348
	30.4.1 Convective Overshooting	349
	30.4.2 Semiconvection	354
30.5	The Schönberg–Chandrasekhar Limit	356
	30.5.1 A Simple Approach: The Virial Theorem and Homology	358
	30.5.2 Integrations for Core and Envelope	360
	30.5.3 Complete Solutions for Stars with Isothermal Cores	361

Part VI Post-Main-Sequence Evolution

31	Evolution Through Helium Burning: Intermediate-Mass Stars	367
31.1	Crossing the Hertzsprung Gap	367
31.2	Central Helium Burning	371
31.3	The Cepheid Phase	375
31.4	To Loop or Not to Loop	378
31.5	After Central Helium Burning	384
32	Evolution Through Helium Burning: Massive Stars	385
32.1	Semiconvection	385
32.2	Overshooting	387
32.3	Mass Loss	389
33	Evolution Through Helium Burning: Low-Mass Stars	391
33.1	Post-Main-Sequence Evolution	391
33.2	Shell-Source Homology	392
33.3	Evolution Along the Red Giant Branch	397
33.4	The Helium Flash	401
33.5	Numerical Results for the Helium Flash	402
33.6	Evolution After the Helium Flash	407
33.7	Evolution from the Zero-Age Horizontal Branch	410

Part VII Late Phases of Stellar Evolution

34	Evolution on the Asymptotic Giant Branch	417
34.1	Nuclear Shells on the Asymptotic Giant Branch	417
34.2	Shell Sources and Their Stability	419

34.3	Thermal Pulses of a Shell Source	422
34.4	The Core-Mass-Luminosity Relation for Large Core Masses	424
34.5	Nucleosynthesis on the AGB	426
34.6	Mass Loss on the AGB	430
34.7	A Sample AGB Evolution	433
34.8	Super-AGB Stars	436
34.9	Post-AGB Evolution	438
35	Later Phases of Core Evolution	439
35.1	Nuclear Cycles	439
35.2	Evolution of the Central Region.....	441
36	Final Explosions and Collapse	449
36.1	The Evolution of the CO-Core	450
36.2	Carbon Ignition in Degenerate Cores	454
36.2.1	The Carbon Flash	454
36.2.2	Nuclear Statistical Equilibrium.....	455
36.2.3	Hydrostatic and Convective Adjustment	458
36.2.4	Combustion Fronts	459
36.2.5	Carbon Burning in Accreting White Dwarfs.....	461
36.3	Collapse of Cores of Massive Stars	461
36.3.1	Simple Collapse Solutions	462
36.3.2	The Reflection of the Infall	465
36.3.3	Effects of Neutrinos	466
36.3.4	Electron-Capture Supernovae.....	469
36.3.5	Pair-Creation Instability.....	469
36.4	The Supernova-Gamma-Ray-Burst Connection	471
 Part VIII Compact Objects		
37	White Dwarfs	475
37.1	Chandrasekhar's Theory	475
37.2	The Corrected Mechanical Structure.....	479
37.2.1	Crystallization	480
37.2.2	Pycnonuclear Reactions.....	482
37.2.3	Inverse β Decays	483
37.2.4	Nuclear Equilibrium	483
37.3	Thermal Properties and Evolution of White Dwarfs	487
38	Neutron Stars	497
38.1	Cold Matter Beyond Neutron Drip	497
38.2	Models of Neutron Stars.....	501
39	Black Holes	509

Part IX Pulsating Stars

40	Adiabatic Spherical Pulsations	519
40.1	The Eigenvalue Problem	519
40.2	The Homogeneous Sphere	523
40.3	Pulsating Polytropes	525
41	Non-adiabatic Spherical Pulsations	529
41.1	Vibrational Instability of the Piston Model	529
41.2	The Quasi-adiabatic Approximation	531
41.3	The Energy Integral	532
	41.3.1 The κ Mechanism	534
	41.3.2 The ε Mechanism	534
41.4	Stars Driven by the κ Mechanism: The Instability Strip	535
41.5	Stars Driven by the ε Mechanism	541
42	Non-radial Stellar Oscillations	543
42.1	Perturbations of the Equilibrium Model	543
42.2	Normal Modes and Dimensionless Variables	545
42.3	The Eigenspectra	548
42.4	Stars Showing Non-radial Oscillations	552

Part X Stellar Rotation

43	The Mechanics of Rotating Stellar Models	557
43.1	Uniformly Rotating Liquid Bodies	557
43.2	The Roche Model	560
43.3	Slowly Rotating Polytropes	562
44	The Thermodynamics of Rotating Stellar Models	565
44.1	Conservative Rotation	565
44.2	Von Zeipel's Theorem	566
44.3	Meridional Circulation	567
44.4	The Non-conservative Case	569
44.5	The Eddington–Sweet Timescale	570
44.6	Meridional Circulation in Inhomogeneous Stars	573
45	The Angular-Velocity Distribution in Stars	575
45.1	Viscosity	575
45.2	Dynamical Stability	577
45.3	Secular Stability	582

References	587
-------------------------	-----

Index	595
--------------------	-----