## **Contents**

Introductionix				
1	J-Filtrations			
	1.1	Introduction	1	
	1.2	Étalé Spaces of Sheaves	2	
	1.3	Étalé Spaces of Sheaves of Ordered Abelian Groups	3	
	1.4	The Category of Pre-J-Filtrations	5	
	1.5	Traces for Étale Maps	10	
	1.6	Pre-J-Filtrations of Sheaves	11	
	1.7	J-Filtrations of Sheaves (the Hausdorff Case)	13	
	1.8	J-Filtered Local Systems (the Hausdorff Case)	16	
	1.9	J-Filtered Local Systems (the Stratified Hausdorff Case)	16	
	1.10	Comments	19	
Part		es-Filtered Local Systems in Dimension One Introduction	23 23	
	2.2	Non-ramified Stokes-Filtered Local Systems	23	
	2.3	Pull-Back and Push-Forward	29	
	2.4	Stokes Filtrations on Local Systems	31	
	2.5	Extension of Scalars	35	
	2.6	Stokes-Filtered Local Systems and Stokes Data	36	
3 Abelianity and		anity and Strictness	39	
	3.1	Introduction	39	
	3.2	Strictness and Abelianity	40	
	3.3	Level Structure of a Stokes-Filtered Local System	41	
	3.4	Proof of Theorem 3.5	43	
	3.5	Proof of the Stability by Extension	48	
	3.6	More on the Level Structure	48	
	3.7	Comments	49	

vi Contents

4	Sto	kes-Perverse Sheaves on Riemann Surfaces	5
	4.1	Introduction	51
	4.2	The Setting	52
	4.3	The Category of Stokes- $\mathbb{C}$ -Constructible Sheaves on $\widetilde{X}$	53
	4.4	Derived Categories and Duality	55
	4.5	The Category of Stokes-Perverse Sheaves on $\widetilde{X}$	58
	4.6	Direct Image to X	60
	4.7	Stokes-Perverse Sheaves on $\widetilde{\underline{X}}$	60
	4.8	Associated Perverse Sheaf on X	62
5	The	e Riemann-Hilbert Correspondence for Holonomic	
		Modules on Curves	65
	5.1	Introduction	65
	5.2	Some Basic Sheaves	66
	5.3	The Riemann–Hilbert Correspondence for Germs	68
	5.4	The Riemann–Hilbert Correspondence in the Global Case	75
	5.5	Compatibility with Duality for Meromorphic Connections	77
6	Δnı	plications of the Riemann–Hilbert Correspondence to	
v		lonomic Distributions	79
	6.1	Introduction	79
	6.2	The Riemann–Hilbert Correspondence for	, ,
	0.2	Meromorphic Connections of Hukuhara–Turrittin Type	80
	6.3	The Hermitian Dual of a Holonomic $\mathcal{D}_X$ -Module	81
	6.4	Asymptotic Expansions of Holonomic Distributions	82
	6.5	Comments	88
_			
7		mann-Hilbert and Laplace on the Affine Line	0.0
		e Regular Case)	89
	7.1	Introduction	89
	7.2	Direct Image of the Moderate de Rham Complex	91
	7.3	Topological Spaces	95
	7.4	Topological Laplace Transform	97
	7.5	Proof of Theorem 7.6 and Compatibility	
			101
	7.6	• • •	103
	7.7	Compatibility of Topological Laplace Transformation	
		•	105
	7.8	Comparison of Both Duality Isomorphisms	108
Par	t II	Dimension Two and More	
8	Res	al Blow-Up Spaces and Moderate de Rham Complexes	15
•	8.1	• •	115
	8.2		116

Contents

	8.3	The Sheaf of Functions with Moderate Growth			
		on the Real Blow-Up Space	118		
	8.4	The Moderate de Rham Complex	121		
	8.5	Examples of Moderate de Rham Complexes	127		
9	Stoke	s-Filtered Local Systems Along a Divisor with			
	Norm	nal Crossings	131		
	9.1	Introduction	131		
	9.2	The Sheaf I on the Real Blow-Up (Smooth Divisor Case)	132		
	9.3	The Sheaf J on the Real Blow-Up (Normal Crossing Case)	133		
	9.4	Goodness	134		
	9.5	Stokes Filtrations on Local Systems	138		
	9.6	Behaviour by Pull-Back	142		
	9.7	Partially Regular Stokes-Filtered Local Systems	145		
10	The Riemann-Hilbert Correspondence for Good				
		omorphic Connections (Case of a Smooth Divisor)	147		
	10.1	Introduction	147		
	10.2	Good Formal Structure of a Meromorphic Connection	149		
	10.3	The Riemann–Hilbert Functor	150		
	10.4	Proof of the Full Faithfulness in Theorem 10.8	152		
	10.5	Elementary and Graded Equivalences	153		
	10.6	Proof of the Essential Surjectivity in Theorem 10.8	155		
11	Good Meromorphic Connections (Formal Theory)				
	11.1	Introduction	159		
	11.2	Preliminary Notation	160		
	11.3	Good Formal Decomposition	161		
	11.4	Good Lattices	166		
	11.5	Proof of Theorem 11.18	168		
	11.6	Comments	175		
12	Good Meromorphic Connections (Analytic Theory) and				
		tiemann-Hilbert Correspondence	177		
	12.1	Introduction	177		
	12.2	Notation	178		
	12.3	The Malgrange–Sibuya Theorem in Higher Dimension	179		
	12.4	The Higher Dimensional Hukuhara–Turrittin Theorem	181		
	12.5	The Riemann–Hilbert Correspondence	188		
	12.6	Application to Hermitian Duality of Holonomic $\mathcal{D}$ -Modules	191		
	12.7	Comments	192		
13	Push-Forward of Stokes-Filtered Local Systems				
	13.1	Introduction	195		
	13.2	Preliminaries	196		
	13.3	Adjunction	197		
	13.4	Recent Advances on Push-Forward and Open Ouestions	198		

viii Contents

	13.5 13.6	An Example of Push-Forward Computation	199
		Leaky Pipes	201
14	Irreg	ular Nearby Cycles	207
	14.1	Introduction	207
	14.2	Moderate Nearby Cycles of Holonomic <i>9</i> -Modules	208
	14.3	Irregular Nearby Cycles (After Deligne)	213
	14.4	Another Proof of the Finiteness Theorem in Dimension Two	214
15	Near	by Cycles of Stokes-Filtered Local Systems	227
	15.1	Introduction	227
	15.2	Nearby Cycles Along a Function (the Good Case)	228
	15.3	Nearby Cycles Along a Function (Dimension Two)	231
	15.4	Comparison	236
Ref	erence	s	239
Ind	ex of n	otation	245
Ind	ex		247