

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
	References	4
<b>2</b>	<b>Theoretical Foundations of Femtosecond Filamentation</b>	<b>7</b>
2.1	The Forward Maxwell Equations	8
2.2	The Nonlinear Optical Response	12
2.2.1	Third-Order Response to a Monochromatic Wave	13
2.2.2	Third-Order Response to an Optical Pulse	14
2.2.3	Plasma Response	16
2.3	Contributions to the Nonlinear Refractive Index	17
2.3.1	Plasma Contributions	17
2.3.2	Contributions Due to the All-Optical Kerr Effect	18
2.4	An Envelope Equation for Few-Cycle Optical Pulses	20
2.4.1	Reduction to the Cylindrically Symmetric Case	23
2.5	Properties of Filamentary Propagation	23
2.5.1	Dispersion	23
2.5.2	Self-Phase Modulation	25
2.5.3	Self-Focusing	26
2.5.4	Modulation Instabilities	28
2.5.5	Space-Time Focusing	29
2.5.6	Intensity Clamping and the Dynamic Spatial Replenishment Model	30
2.5.7	Pulse Self-Compression	31
	References	31
<b>3</b>	<b>Pulse Self-Compression in Femtosecond Filaments</b>	<b>35</b>
3.1	The Self-Pinching Mechanism: Self-Compression as a Spatial Effect	38
3.2	Stationary Solutions Beyond the Variational Approach	45
3.3	Cascaded Self-Compression	51

3.3.1	Experimental Evidence of Cascaded Self-Compression . .	57
3.4	Temporal Self-Restoration in Femtosecond Filaments . . . . .	60
3.4.1	Experimental Prerequisites . . . . .	62
3.4.2	Experiment 1: Variation of Window Position . . . . .	64
3.4.3	Experiment 2: Windowless Measurement . . . . .	66
3.4.4	Comparison with Numerical Simulations . . . . .	67
	References . . . . .	74
<b>4</b>	<b>Saturation and inversion of the all-optical Kerr effect . . . . .</b>	<b>79</b>
4.1	Kramers-Kronig Relations in Linear and Nonlinear Optics . . . .	81
4.2	Ionization of Atoms in Intense Laser Fields . . . . .	82
4.2.1	Keldysh Theory and Its Generalizations . . . . .	84
4.2.2	A Recent Modification of the PPT Model . . . . .	90
4.2.3	The Multiphoton Limit . . . . .	91
4.3	Kramers-Kronig Approach to Second Order Nonlinear Refraction . . . . .	92
4.4	Higher Order Kerr Effect and Femtosecond Filamentation . . . .	99
	References . . . . .	106
<b>5</b>	<b>Conclusions . . . . .</b>	<b>111</b>
	References . . . . .	113
	<b>Appendix A: The Nonlinear Schrödinger Equation . . . . .</b>	<b>115</b>
	<b>Appendix B: Numerical Method . . . . .</b>	<b>119</b>
	<b>Appendix C: Characterization of Ultrashort Few-Cycle Pulses . . . . .</b>	<b>123</b>