

Contents

- 1 Linear Equations 1**
 - 1.1 Linear Equations and Functions 1
 - 1.2 Gaussian Elimination 6
 - 1.3 Examples* 15
- 2 Matrices and Determinants 25**
 - 2.1 Determinants of Orders 2 and 3 25
 - 2.2 Determinants of Arbitrary Order 30
 - 2.3 Properties that Characterize Determinants 37
 - 2.4 Expansion of a Determinant Along Its Columns 39
 - 2.5 Cramer’s Rule 42
 - 2.6 Permutations, Symmetric and Antisymmetric Functions 44
 - 2.7 Explicit Formula for the Determinant 50
 - 2.8 The Rank of a Matrix 53
 - 2.9 Operations on Matrices 60
 - 2.10 Inverse Matrices 70
- 3 Vector Spaces 79**
 - 3.1 The Definition of a Vector Space 79
 - 3.2 Dimension and Basis 86
 - 3.3 Linear Transformations of Vector Spaces 101
 - 3.4 Change of Coordinates 107
 - 3.5 Isomorphisms of Vector Spaces 112
 - 3.6 The Rank of a Linear Transformation 118
 - 3.7 Dual Spaces 120
 - 3.8 Forms and Polynomials in Vectors 127
- 4 Linear Transformations of a Vector Space to Itself 133**
 - 4.1 Eigenvectors and Invariant Subspaces 133
 - 4.2 Complex and Real Vector Spaces 142
 - 4.3 Complexification 149
 - 4.4 Orientation of a Real Vector Space 154

5	Jordan Normal Form	161
5.1	Principal Vectors and Cyclic Subspaces	161
5.2	Jordan Normal Form (Decomposition)	165
5.3	Jordan Normal Form (Uniqueness)	169
5.4	Real Vector Spaces	173
5.5	Applications*	176
6	Quadratic and Bilinear Forms	191
6.1	Basic Definitions	191
6.2	Reduction to Canonical Form	198
6.3	Complex, Real, and Hermitian Forms	204
7	Euclidean Spaces	213
7.1	The Definition of a Euclidean Space	213
7.2	Orthogonal Transformations	223
7.3	Orientation of a Euclidean Space*	230
7.4	Examples*	233
7.5	Symmetric Transformations	245
7.6	Applications to Mechanics and Geometry*	255
7.7	Pseudo-Euclidean Spaces	265
7.8	Lorentz Transformations	275
8	Affine Spaces	289
8.1	The Definition of an Affine Space	289
8.2	Affine Spaces	294
8.3	Affine Transformations	301
8.4	Affine Euclidean Spaces and Motions	309
9	Projective Spaces	319
9.1	Definition of a Projective Space	319
9.2	Projective Transformations	328
9.3	The Cross Ratio	335
9.4	Topological Properties of Projective Spaces*	339
10	The Exterior Product and Exterior Algebras	349
10.1	Plücker Coordinates of a Subspace	349
10.2	The Plücker Relations and the Grassmannian	353
10.3	The Exterior Product	358
10.4	Exterior Algebras*	367
10.5	Appendix*	374
11	Quadrics	385
11.1	Quadrics in Projective Space	385
11.2	Quadrics in Complex Projective Space	394
11.3	Isotropic Subspaces	398
11.4	Quadrics in a Real Projective Space	410
11.5	Quadrics in a Real Affine Space	414
11.6	Quadrics in an Affine Euclidean Space	425
11.7	Quadrics in the Real Plane*	428

12 Hyperbolic Geometry 433

12.1 Hyperbolic Space* 434

12.2 The Axioms of Plane Geometry* 443

12.3 Some Formulas of Hyperbolic Geometry* 454

13 Groups, Rings, and Modules 467

13.1 Groups and Homomorphisms 467

13.2 Decomposition of Finite Abelian Groups 475

13.3 The Uniqueness of the Decomposition 481

13.4 Finitely Generated Torsion Modules over a Euclidean Ring* 484

14 Elements of Representation Theory 497

14.1 Basic Concepts of Representation Theory 497

14.2 Representations of Finite Groups 503

14.3 Irreducible Representations 508

14.4 Representations of Abelian Groups 511

Historical Note 515

References 517

Index 521