

Contents

List of Abbreviations	XXI
1 Mathematical Signs and Symbols	1
1.1 Pragmatic Signs	1
1.2 General Arithmetic Relations and Links	1
1.3 Sets of Numbers	2
1.4 Special Numbers and Links	3
1.5 Limit	3
1.6 Exponential Functions, Logarithm	4
1.7 Trigonometric Functions, Hyperbolic Functions	4
1.8 Vectors, Matrices	5
1.9 Sets	6
1.10 Relations	7
1.11 Functions	7
1.12 Order Structures	7
1.13 SI Multiplying and Dividing Prefixes	8
1.14 Greek Alphabet	9
	IX

2	Logic	11
2.1	Mathematical Logic	11
2.2	Propositional Logic	11
2.2.1	Propositional Variable	11
2.2.2	Truth Tables	12
3	Arithmetic	15
3.1	Sets	15
3.1.1	General	15
3.1.2	Set Relations	16
3.1.3	Set Operations	17
3.1.4	Relations, Laws, Rules of Calculation for Sets	19
3.1.5	Intervals	21
3.1.6	Numeral Systems	22
3.1.6.1	Decimal System (Decadic System)	23
3.1.6.2	Dual System (Binary System)	23
3.1.6.3	Roman Numeral System	24
3.2	Elementary Calculus	24
3.2.1	Elementary Foundations	24
3.2.1.1	Axioms	25
3.2.1.2	Factorisation	25
3.2.1.3	Relations	26
3.2.1.4	Absolute Value, Signum	26
3.2.1.5	Fractions	27
3.2.1.6	Polynomial Division	27
3.2.1.7	Horner's Scheme (Horner's Method)	29
3.2.2	Conversions of Terms	30
3.2.2.1	Binomial Formulas	30
3.2.2.2	Binomial Theorem	31
3.2.2.3	General Binomial Theorem for Nat- ural Exponents	31
3.2.2.4	General Binomial Theorem for Real Exponents	31
3.2.2.5	Polynomial Terms	32
3.2.3	Summation and Product Notation	32
3.2.3.1	Summation Notation	32

	3.2.3.2	Product Notation	33
	3.2.4	Powers, Roots	34
	3.2.5	Logarithms	37
	3.2.6	Factorial	39
	3.2.7	Binomial Coefficient	40
3.3		Sequences	41
	3.3.1	Definition	41
	3.3.2	Limit of a Sequence	44
	3.3.3	Arithmetic and Geometric Sequences	46
3.4		Series	47
	3.4.1	Definition	47
	3.4.2	Arithmetic and Geometric Series	47
4		Algebra	51
4.1		Fundamental Terms	51
4.2		Linear Equations	53
	4.2.1	Linear Equations with One Variable	53
	4.2.2	Linear Inequations with One Variable	56
	4.2.3	Linear Equations with Multiple Variables ...	56
	4.2.4	Systems of Linear Equations	57
	4.2.5	Linear Inequations with Multiple Variables .	61
4.3		Non-linear Equations	62
	4.3.1	Quadratic Equations with One Variable	62
	4.3.2	Cubic Equations with One Variable	65
	4.3.3	Biquadratic Equations	67
	4.3.4	Equations of the n^{th} Degree	68
	4.3.5	Radical Equations	69
4.4		Transcendental Equations	71
	4.4.1	Exponential Equations	71
	4.4.2	Logarithmic Equations	73
4.5		Approximation Methods	75
	4.5.1	Regula falsi (Secant Method)	75
	4.5.2	Newton's Method (Tangent Method)	77

4.5.3	General Approximation Method (Fixed-point Iteration)	80
5	Linear Algebra	87
5.1	Fundamental Terms	87
5.1.1	Matrix	87
5.1.2	Equality/Inequality of Matrices	88
5.1.3	Transposed Matrix	89
5.1.4	Vector	89
5.1.5	Special Matrices and Vectors	92
5.2	Operations with Matrices	94
5.2.1	Addition of Matrices	94
5.2.2	Multiplication of Matrices	96
5.2.2.1	Multiplication of a Matrix with a Scalar	96
5.2.2.2	The Scalar Product of Two Vectors	98
5.2.2.3	Multiplication of a Matrix by a Column Vector	100
5.2.2.4	Multiplication of a Row Vector by a Matrix	102
5.2.2.5	Multiplication of Two Matrices	103
5.3	The Inverse of a Matrix	107
5.3.1	Introduction	107
5.3.2	Determination of the Inverse with the Usage of the Gaussian Elimination Method	109
5.4	The Rank of a Matrix	113
5.4.1	Definition	113
5.4.2	Determination of the Rank of a Matrix	113
5.5	The Determinant of a Matrix	117
5.5.1	Definition	117
5.5.2	Calculation of Determinants	118
5.5.3	Characteristics of Determinants	124

5.6	The Adjoint of a Matrix	125
5.6.1	Definition	125
5.6.2	Determination of the Inverse with the Usage of the Adjoint	127
6	Combinatorics	129
6.1	Introduction	129
6.2	Permutations	133
6.3	Variations	135
6.4	Combinations	136
7	Financial Mathematics	141
7.1	Calculation of Interest	141
7.1.1	Fundamental Terms	141
7.1.2	Annual Interest	142
7.1.2.1	Simple Interest Calculation	142
7.1.2.2	Compound Computation of Inter- est	144
7.1.2.3	Composite Interest	146
7.1.3	Interest During the Period	158
7.1.3.1	Simple Interest Calculation (linear)	159
7.1.3.2	Simple Interest Using the Nomi- nal Annual Interest Rate	159
7.1.3.3	Compound Interest (exponential) .	160
7.1.3.4	Interest with Compound Interest Using a Conforming Annual Inter- est Rate	161
7.1.3.5	Mixed Interest	162
7.1.3.6	Steady Interest Rate	163
7.2	Annual Percentage Rate	168
7.3	Depreciation	173
7.3.1	Time Depreciation	173
7.3.1.1	Linear Depreciation	173

	7.3.1.2	Arithmetic-Degressive Depreciation	174
	7.3.1.3	Geometric-Degressive Depreciation	176
7.3.2		Units of Production Depreciation	178
7.3.3		Extraordinary Depreciation	179
7.4		Annuity Calculation	180
7.4.1		Fundamental Terms	180
7.4.2		Finite, Regular Annuity	183
	7.4.2.1	Annual Annuity with Annual Interest	183
	7.4.2.2	Annual Annuity with Sub-Annual Interest	187
	7.4.2.3	Sub-Annual Annuity with Annual Interest	190
	7.4.2.4	Sub-Annual Annuity with Sub-Annual Interest	194
7.4.3		Finite, Variable Annuity	213
	7.4.3.1	Irregular Annuity	213
	7.4.3.2	Arithmetic Progressive Annuity ...	220
	7.4.3.3	Geometric Progressive Annuity ...	231
7.4.4		Perpetuity	234
7.5		Sinking Fund Calculation	235
7.5.1		Fundamental Terms	236
7.5.2		Annuity Repayment	238
7.5.3		Repayment by Instalments	241
7.5.4		Repayment with Premium	243
	7.5.4.1	Annuity Repayment with Premium	243
	7.5.4.2	Repayment of an Instalment Debt with Premium	248
7.5.5		Repayment with Discount (Disagio)	249
	7.5.5.1	Annuity Repayment with Discount when Immediately Booked as Interest Expense	251
	7.5.5.2	Annuity Repayment with Discount when a Disagio is Included in Pre-paid Expenses	253
	7.5.5.3	Instalment Repayment with Discount when Immediately Booked as Interest Expense	253

	7.5.5.4	Instalment Repayment with Discount when a Disagio is Included in Prepaid Expenses	254
7.5.6		Grace Periods	255
7.5.7		Rounded Annuities	257
	7.5.7.1	Percentage Annuity	257
	7.5.7.2	Repayment of Bonds	260
7.5.8		Repayment During the Year	266
	7.5.8.1	Annuity Repayment During the Year	266
	7.5.8.2	Repayment by Instalments During the Year	274
7.6		Investment Calculation	279
	7.6.1	Fundamental Terms	280
	7.6.2	Fundamentals of Financial Mathematics ...	283
	7.6.3	Methods of Static Investment Calculation ..	286
	7.6.4	Methods of Dynamic Investment Calculation	286
	7.6.4.1	Net Present Value Method (Net Present Value, Amount of Capital, Final Asset Value)	287
	7.6.4.2	Annuity Method	290
	7.6.4.3	Internal Rate of Return Method ..	293
8		Optimisation of Linear Models	297
8.1		Lagrange Method	297
	8.1.1	Introduction	297
	8.1.2	Formation of the Lagrange Function	297
	8.1.3	Determination of the Solution	298
	8.1.4	Interpretation of λ	301
	8.1.5	Identification of the Type of Optimum	302
8.2		Linear Optimisation	313
	8.2.1	Introduction	313
	8.2.2	The Linear Programming Approach	313
	8.2.3	Graphical Solution	314
	8.2.4	Primal Simplex Algorithm	317
	8.2.5	Simplex Tableau (Basic Structure)	318

8.2.6	Dual Simplex Algorithm	324
8.3	Non-linear Optimisation	333
8.3.1	Introduction	333
8.3.2	Basic Properties of Non-linear Optimisation	334
8.3.3	Non-linear Optimisation Methods	336
8.3.3.1	Search Strategies.....	337
8.3.3.2	Deterministic Search Strategies ..	337
8.3.3.3	The Nelder-Mead Simplex Search Procedure	338
8.3.4	Conclusion	346
9	Functions	349
9.1	Introduction	349
9.1.1	Composition of Functions	353
9.1.2	Inverse Function	355
9.2	Classification of Functions	357
9.2.1	Rational Functions	358
9.2.1.1	Polynomial Functions	358
9.2.1.2	Broken Rational Functions	358
9.2.2	Non-rational Functions	362
9.2.2.1	Power Functions.....	362
9.2.2.2	Root Function	365
9.2.2.3	Transcendental Functions.....	366
9.2.2.3.1	Exponential Functions ..	366
9.2.2.3.2	Logarithmic Functions ..	372
9.2.2.4	Trigonometric Functions (Angle Func- tions/Circular Functions)	378
9.3	Characteristics of Real Functions	406
9.3.1	Boundedness	406
9.3.2	Symmetry	408
9.3.2.1	Axial Symmetry	408
9.3.2.2	Point Symmetry	410
9.3.3	Transformations	413
9.3.3.1	Vertex Form	415
9.3.4	Continuity	418
9.3.5	Infinite Discontinuities	418

9.3.6	Removable Discontinuities	420
9.3.7	Jump Discontinuities	421
9.3.8	Homogeneity	422
9.3.9	Periodicity	423
9.3.10	Zeros	423
9.3.11	Local Extremes	424
9.3.12	Monotonicity	425
9.3.13	Concavity and Convexity Inflection Points .	426
9.3.14	Asymptotes	428
	9.3.14.1 Horizontal Asymptotes	429
	9.3.14.2 Vertical Asymptote	431
	9.3.14.3 Oblique Asymptote	432
	9.3.14.4 Asymptotic Curve	433
9.3.15	Tangent Lines to a Curve	434
9.3.16	Normal Lines to a Curve	435
9.4	Exercises	436
10	Differential Calculus	441
10.1	Differentiation of Functions with One Independent Variable	441
	10.1.1 General	441
	10.1.2 First Derivative of Elementary Functions ...	444
	10.1.3 Derivation Rules	446
	10.1.4 Higher Derivations	448
	10.1.5 Differentiation of Functions with Parameters	449
	10.1.6 Curve Sketching	449
10.2	Differentiation of Functions with More Than One Independent Variable	459
	10.2.1 Partial Derivatives (1 st Order)	459
	10.2.2 Partial Derivatives (2 nd Order)	462
	10.2.3 Local Extrema of the Function $f = f(x, y)$..	464
	10.2.3.1 Relative Extrema without Constraint of the Function $f = f(x, y)$	464

10.2.3.2	Relative Extrema with m Constraints of the Function $f = f(x_1, \dots, x_n)$ with $m < n$	473
10.2.4	Differentials of the Function $f = f(x_1, \dots, x_n)$	477
10.3	Theorems of Differentiable Functions	479
10.3.1	Mean Value Theorem for Differential Calculus	479
10.3.2	Generalized Mean Value Theorem for Differential Calculus	480
10.3.3	Rolle's Theorem	480
10.3.4	L'Hospital's Rule	481
10.3.5	Bounds Theorem for Differential Calculus ..	482
11	Integral Calculus	483
11.1	Introduction	483
11.2	The Indefinite Integral	484
11.2.1	Definition/Determining the Antiderivative ...	484
11.2.2	Elementary Calculation Rules for the Indefinite Integral	487
11.3	The Definite Integral	488
11.3.1	Introduction	488
11.3.2	Relationship between the Definite and the Indefinite Integral	492
11.3.3	Special Techniques of Integration	497
11.3.3.1	Partial Integration	497
11.3.3.2	Integration by Substitution	499
11.4	Multiple Integrals	500
11.5	Integral Calculus and Economic Problems	501
11.5.1	Cost Functions	501
11.5.2	Revenue Function (= Sales Function)	503
11.5.3	Profit Functions	504

Contents	XIX
12 Elasticities	509
12.1 Definition of Elasticity	509
12.2 Arc Elasticity	510
12.3 Point Elasticity	514
12.4 Price Elasticity of Demand ϵ_{xp}	517
12.5 Cross Elasticity of Demand ϵ_{xAPB}	522
12.6 Income Elasticity of Demand ϵ_{xy}	524
13 Economic Functions	525
13.1 Supply Function	525
13.2 Demand Function / Inverse Demand Function	527
13.3 Market Equilibrium	529
13.4 Buyer's Market and Seller's Market	530
13.5 Supply Gap	531
13.6 Demand Gap	531
13.7 Revenue Function	533
13.8 Cost Functions	539
13.9 Neoclassical Cost Function	547
13.10 Cost Function According to the Law of Diminishing Returns	555
13.11 Direct Costs versus Indirect Costs	568
13.11.1 One-Dimensional Cost Allocation Principles	571
13.11.2 Multi-Dimensional Cost Allocation Principles	573

XX	Contents
13.12 Profit Function	576
14 The Peren Theorem	
The Mathematical Frame in Which We Live	585
A Financial Mathematical Factors	593
B Bibliography	639
Index	649