

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	A General Background	1
1.2	Text Preview	5
	References	7
<b>2</b>	<b>Stability Loss Problems Related to Solid and Hollow Circular Cylinders</b>	<b>9</b>
2.1	Formulation of the Problem Related to the Global Stability Loss	9
2.2	Method of Solution for the Global Stability Loss Problem	13
2.3	Approximate Equations for the Stability Loss of the Cylinder-Beam Obtained from Equations of the TDLTS by the Average-Integrating Procedure	26
2.3.1	Bernoulli Beam theory	26
2.4	The Third Order Refined Beam Theory	30
2.5	Numerical Results and Discussions	32
2.5.1	Solid Cylinder	35
2.5.2	Hollow Cylinder	41
2.6	Formulation of the Problem Related to the Rotationally Symmetric Local Stability Loss	45
2.7	Method of Solution for the Rotationally Symmetric Problem	48
2.8	Approximate Equations of the Stability Loss of the Cylinder-Shell Obtained from Equations of the TDLTS by the Average-Integrating Procedure	55
2.8.1	Kirchhoff-Love Shell Theory	55
2.9	The Third Order Refined Shell Theory	59
2.10	Numerical Results Related to the Rotationally Symmetric Stability Loss Problem	61
2.11	Conclusions	68
	References	69

<b>3</b>	<b>Stability Loss Problems for Viscoelastic Plates</b>	<b>71</b>
3.1	Formulation of the Problem and Basic Field Equations	71
3.2	Approach for the Solution to the Stability Loss Problem for Rectangular Plate	74
3.3	Simply Supported Rectangular Plate.	80
3.3.1	Deriving Approximate Equations of the Stability Loss of the Simply Supported Rectangular Plate from Equations of the TDLTS by the Average-Integrating Procedure.	80
3.3.2	Solution for the Formulated Mathematical Problems for the Simply Supported Plate	88
3.3.3	Numerical Results and Discussions	90
3.4	Rectangular Plate Clamped at Two Opposite Ends and Simply Supported at the Two Other Opposite Ends	94
3.4.1	Solution Procedure of the Equations of the TDLTS. Semi-Analytical FEM Modeling	94
3.4.2	Solution Procedure for the Approximate Stability Loss Equations	98
3.4.3	Numerical Results and Discussions	99
3.5	Rectangular Plate Clamped at all Ends.	102
3.5.1	Solution Procedure for the TDLTS Problem. 3D FEM Modeling.	102
3.5.2	Solution Procedure for the Approximate Stability Loss Problems	104
3.5.3	Numerical Results and Discussions	105
3.6	Symmetric Stability Loss of the Circular Plate	107
3.6.1	Formulation of the Problem and Governing Field Equations	107
3.6.2	Method of Solution	110
3.6.3	Numerical Results and Discussions	117
3.7	Stability Loss of the Rotating Circular and Annular Discs	125
3.7.1	Formulation of the Problem.	125
3.7.2	Method of Solution	127
3.7.3	Numerical Results and Discussions	129
	References	132
<b>4</b>	<b>Buckling Delamination of Elastic and Viscoelastic Composite Plates with Cracks</b>	<b>135</b>
4.1	Background of Related Problems.	135
4.2	Buckling Delamination Problems for Plate-Strips with a Single Crack	137
4.2.1	Formulation of the Problems.	137
4.2.2	Method of Solution	140
4.2.3	Numerical Results and Discussions	146

- 4.3 Buckling Delamination of the Plate-Strip with Two Parallel Cracks . . . . . 153
  - 4.3.1 Mathematical Formulation of the Problem . . . . . 153
  - 4.3.2 Method of Solution: FEM Modeling. . . . . 155
  - 4.3.3 Numerical Results and Discussions . . . . . 156
- 4.4 Buckling Delamination of the Plate-Strip with Two Collinear Cracks . . . . . 159
  - 4.4.1 Formulation of the Problem and Solution Method . . . . 159
  - 4.4.2 Numerical Results and Discussions . . . . . 161
- 4.5 Buckling Delamination of the Three-Layered (Sandwich) Plate-Strip with Two Parallel Interface Cracks. . . . . 164
  - 4.5.1 Formulation of the Problem. . . . . 164
  - 4.5.2 Method of Solution . . . . . 166
  - 4.5.3 Numerical Results and Discussions . . . . . 172
- 4.6 Buckling Delamination of the Three-Layered (Sandwich) Plate-Strip with Two Collinear Interface Cracks . . . . . 176
  - 4.6.1 Formulation of the Problem and Method of Solution . . . . . 176
  - 4.6.2 Numerical Results and Discussions . . . . . 179
- 4.7 Buckling Delamination of the Elastic and Viscoelastic Composite Circular Plate-Disc with a Penny-Shaped Crack. . . . 181
  - 4.7.1 Formulation of the Problem. . . . . 181
  - 4.7.2 Method of Solution . . . . . 183
  - 4.7.3 Numerical Results and Discussions . . . . . 186
- 4.8 Buckling Delamination of the Three-Layered (Sandwich) Circular Plate-Disc with Two Parallel Interface Penny-Shaped Cracks . . . . . 190
  - 4.8.1 Formulation of the Problem. . . . . 190
  - 4.8.2 Method of Solution . . . . . 193
  - 4.8.3 Numerical Results and Discussions . . . . . 200
- 4.9 Remarks on the FEM Modeling of the Crack's Tips . . . . . 202
- 4.10 Buckling Delamination of a Rectangular Plate Containing a Rectangular Crack. . . . . 205
  - 4.10.1 Formulation of the Problems . . . . . 206
  - 4.10.2 Solution Method . . . . . 209
  - 4.10.3 FEM Modeling . . . . . 212
  - 4.10.4 Numerical Results and Discussions . . . . . 213
- 4.11 Buckling Delamination of a Sandwich Rectangular Plate with Interface Rectangular Cracks . . . . . 231
  - 4.11.1 Formulation of the Problem. . . . . 231
  - 4.11.2 Solution Method . . . . . 237
  - 4.11.3 FEM Modeling . . . . . 241
  - 4.11.4 Numerical Results and Discussions . . . . . 242
- References . . . . . 265

<b>5</b>	<b>Surface and Internal Stability Loss in the Structure of Elastic and Viscoelastic Layered Composites. . . . .</b>	<b>269</b>
5.1	Background of Related Problems. . . . .	269
5.2	Stability Loss in the Structure of Elastic and Viscoelastic Layered Composites with Periodical Initial Imperfections. . . . .	271
5.2.1	Formulation of the Problem on the Determination of the Stress–Strain State in a Layered Composite with an Arbitrary Number of Layers with Initially Infinitesimal Imperfections . . . . .	271
5.2.2	Method of Solution . . . . .	273
5.2.3	Numerical Results and Discussions . . . . .	288
5.3	Stability Loss in the Structure of the Elastic and Viscoelastic Layered Composites with Local Initial Imperfections. . . . .	294
5.3.1	Formulation of the Problem and Method of Solution . . . . .	294
5.3.2	Numerical Results and Discussions . . . . .	296
5.4	The Influence of the Inclination of the Local Initial Imperfections of the Reinforcing Layers on the Values of the Critical Parameters . . . . .	302
5.4.1	Formulation of the Problem and Solution Method . . . . .	302
5.4.2	Results and Discussions . . . . .	306
5.4.3	Conclusions. . . . .	308
5.5	Surface Undulation Instability of the Viscoelastic Half-Space Covered with a Stack of Layers in Bi-Axial Compression. . . . .	309
5.5.1	Formulation of the Problem. . . . .	310
5.5.2	Method of Solution . . . . .	312
5.5.3	Numerical Results and Discussions . . . . .	319
	References . . . . .	334
<b>6</b>	<b>Stability Loss in the Structure of Unidirected Fibrous Elastic and Viscoelastic Composites . . . . .</b>	<b>337</b>
6.1	Some General Remarks on the Field Equations, Problem Formulations and Method of Solution. . . . .	337
6.1.1	General Remarks on the Field Equations and Problem Formulations . . . . .	337
6.1.2	General Remarks on the Method of Solution. . . . .	341
6.2	Micro Buckling of a Single Fiber in the Viscoelastic Matrix. . . . .	348
6.2.1	Formulation of the Problem and Method of Solution . . . . .	348
6.2.2	Numerical Results and Discussions . . . . .	350
6.3	Internal Stability Loss of Two Neighboring Fibers in a Viscoelastic Matrix . . . . .	353
6.3.1	Formulation of the Problem and Method of Solution . . . . .	353
6.3.2	Numerical Results and Discussion . . . . .	362
6.4	Internal Stability Loss of a Row of Unidirected Periodically	

Located Fibers in a Viscoelastic Matrix . . . . .	368
6.4.1 Formulation of the Problem and Solution Method . . . .	368
6.4.2 Numerical Results and Discussions . . . . .	373
6.5 Stability Loss of a Micro-Fiber in an Elastic and a Viscoelastic Matrix Near the Free Convex Cylindrical Surface . . . . .	374
6.5.1 Formulation of the Problem. . . . .	374
6.5.2 Method of Solution . . . . .	377
6.5.3 Numerical Results and Discussions . . . . .	391
References . . . . .	399
<b>Supplement 1: Applications of the Approach Developed in Chap. 4 on the Problems Related to the Stress Concentration in Initially Stressed Bodies . . . . .</b>	<b>401</b>
<b>Supplement 2: Self-Balanced Stresses Caused by Periodical Curving of Two Neighbouring and Periodically Located Row of Fibers in an Infinite Matrix . . . . .</b>	<b>425</b>
<b>Index . . . . .</b>	<b>441</b>