Contents

Preface — VII

Acknowledgments —— IX

| 1 | Introduction to ideal numbers —— 1 |
|-----|--|
| 1.1 | Discriminants and principal polynomials —— 1 |
| 1.2 | Definition of ideal numbers —— 3 |
| 1.3 | Ideal numbers and quadratic polynomials —— 5 |
| 1.4 | Ideal numbers and quadratic forms —— 9 |
| 1.5 | Ideal numbers and quadratic numbers —— 12 |
| 1.6 | Ideal numbers and ideals in quadratic domains —— 18 |
| 2 | Congruence classes of ideal numbers —— 21 |
| 2.1 | Congruence and divisibility of ideal numbers —— 21 |
| 2.2 | Ideal numbers of prime power norm and level zero —— 25 |
| 2.3 | Seeding maps —— 30 |
| 2.4 | Counting congruence classes of ideal numbers —— 34 |
| 3 | Composition of ideal numbers —— 39 |
| 3.1 | Composition of congruence classes —— 39 |
| 3.2 | Prime subgroups of level zero —— 43 |
| 3.3 | Prime kernels of a discriminant —— 45 |
| 3.4 | The kernel of a prime subgroup —— 48 |
| 3.5 | Level subgroups —— 51 |
| 3.6 | Prime subgroups of positive level —— 54 |
| 4 | Classes of ideal numbers —— 60 |
| 4.1 | Equivalence of ideal numbers —— 60 |
| 4.2 | Proper representations by ideal numbers —— 64 |
| 4.3 | Reduction of ideal numbers —— 67 |
| 4.4 | Automorphs of ideal numbers —— 71 |
| 4.5 | The class group of a discriminant —— 74 |
| 5 | Genera of ideal numbers —— 78 |
| 5.1 | The genus group of a discriminant —— 78 |
| 5.2 | Genus equivalence of ideal numbers —— 84 |
| 5.3 | The existence of genera of ideal numbers —— 88 |
| 5.4 | Class groups and genera —— 92 |



| 6 | Ideal numbers of negative discriminant —— 96 |
|-------------|--|
| 6.1 | Reduced ideal numbers of negative discriminant —— 96 |
| 6.2 | Reduced ideal numbers and equivalence — 100 |
| 6.3 | Class groups of primitive negative discriminant —— 103 |
| 6.4 | Class groups and representations — 106 |
| 6.5 | Negative Boolean discriminants —— 111 |
| 6.6 | Class groups of arbitrary negative discriminant —— 115 |
| 7 | Ideal numbers of positive discriminant —— 120 |
| 7.1 | Reduced ideal numbers of positive discriminant —— 120 |
| 7.2 | Reduction maps —— 124 |
| 7.3 | Periodicity of reduction maps —— 129 |
| 7.4 | Reduction and equivalence —— 133 |
| 7.5 | The continued fraction of an indefinite ideal number —— 137 |
| 7.6 | Representations of integers by indefinite ideal numbers —— 141 |
| 7.7 | Class group calculations —— 143 |
| 4 | Appendix: Review of number theory —— 149 |
| 4.1 | Divisibility and congruence —— 149 |
| A.2 | Combinations —— 151 |
| 4.3 | Prime factorization —— 153 |
| 4.4 | The Euclidean algorithm —— 155 |
| A. 5 | Linear equations and congruences —— 157 |
| ۸.6 | Systems of linear congruences —— 160 |
| ٦.7 | Quadratic congruences modulo primes —— 162 |
| 8. | Quadratic reciprocity theorem —— 167 |
| 3 | Appendix: Review of groups —— 172 |
| 3.1 | Groups and subgroups —— 172 |
| 3.2 | Cosets and quotient groups —— 175 |
| 3.3 | Finite Abelian groups —— 177 |
| : | Appendix: Ideals of quadratic domains —— 185 |
| . .1 | Ideals of integral domains —— 185 |
| . .2 | Quadratic domains —— 187 |
| 3 | Classification of ideals of a quadratic domain —— 189 |
| 4 | Ideal number expressions for ideals —— 193 |
| . .5 | Complete quadratic domains and their subdomains —— 195 |
| .6 | Multiplication of ideals of a quadratic domain —— 198 |
| 7 | Composition of primitive ideals —— 203 |
| 3.3 | Equivalence of ideals —— 206 |
| .9 | Ordered bases of ideals —— 207 |

| C.10 | Ideal equivalence and ideal number equivalence —— 212 | |
|-------------------|--|--|
| D | Appendix: Continued fractions —— 215 | |
| D.1 | Continued fractions of rational numbers —— 215 | |
| D.2 | Convergents —— 217 | |
| D.3 | Infinite continued fractions —— 220 | |
| D.4 | Calculating periodic continued fractions —— 222 | |
| D.5 | Continued fractions and rational approximations —— 225 | |
| References —— 233 | | |
| Index - | 235 | |