# Contents

**Preface — V**

**1 Physical backgrounds for some nonlinear evolution equations — 1**
1.1 The wave equation under weak nonlinear action and KdV equation — 1
1.2 Zakharov equation and the solitons in plasma — 9
1.3 Landau–Lifshitz equation and the magnetized motion — 18
1.4 Boussinesq equation, Toda Lattice and Born–Infeld equation — 21
1.5 2D K-P equation — 25

**2 The properties of the solutions for some nonlinear evolution equations — 29**
2.1 The smooth solution for the initial-boundary value problem of nonlinear Schrödinger equation — 30
2.2 The existence of the weak solution for the initial-boundary value problems of generalized Landau–Lifshitz equations — 34
2.2.1 The basic estimates of the linear parabolic equations — 34
2.2.2 The existence of the spin equations — 35
2.2.3 The existence of the solution to the initial-boundary value problem of the generalized Landau–Lifshitz equations — 38
2.3 The large time behavior for generalized KdV equations — 42
2.4 The decay estimates for the weak solution of Navier–Stokes equations — 59
2.5 The "blowing up" phenomenon for the Cauchy problem of nonlinear Schrödinger equations — 70
2.6 The "blow up" problem for the solutions of some semilinear and hyperbolic equations — 76
2.7 The smoothness of the weak solutions for Benjamin–Ono equations — 90

**3 Some results from the study of some nonlinear evolution equations — 103**
3.1 Nonlinear wave equations and nonlinear Schrödinger equations — 103
3.2 KdV equations — 119
3.3 Landau–Lifshitz equations — 129

**4 Similarity solution and the Painlevé property for some nonlinear evolution equations — 139**
4.1 Classical infinitesimal transformations — 139
4.2 Structure of Lie algebra for infinitesimal operator — 153
4.3 Nonclassical infinitesimal transformations — 155

Bibliografische Informationen
http://d-nb.info/1190750252

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

4.4 A direct method for solving similarity solutions — 160
4.5 The Painlevé properties for some PDE — 170

**5 Infinite-dimensional dynamical systems — 179**
5.1 Infinite-dimensional dynamical systems — 180
5.2 Some problems for infinite-dimensional dynamical systems — 184
5.3 Global attractor and its Hausdorff, fractal dimensions — 193
5.4 Global attractor and the bounds of Hausdorff dimensions for weak damped KdV equation — 203
5.4.1 Uniform a priori estimation with respect to $t$ — 203
5.5 Global attractor and the bounds of Hausdorff dimensions for weak damped nonlinear Schrödinger equation — 214
5.5.1 Uniform a priori estimation with respect to $t$ — 214
5.5.2 Transforming to Cauchy problem of the operator — 219
5.5.3 The existence of bounded absorbing set of $H^1$ modular — 221
5.5.4 The existence of bounded absorbing set of $H^2$ modular — 222
5.5.5 Nonlinear semigroup and long-time behavior — 226
5.5.6 The dimension of invariant set — 229
5.6 Global attractor and the bounds of Hausdorff, fractal dimensions for damped nonlinear wave equation — 235
5.6.1 Linear wave equation — 235
5.6.2 Nonlinear wave equation — 240
5.6.3 The maximal attractor — 247
5.6.4 Dimension of the maximal attractor — 250
5.6.5 Application — 257
5.6.6 Nonautonomous system — 262
5.7 Inertial manifold for one class of nonlinear evolution equations — 265
5.8 Approximate inertial manifold — 284
5.9 Nonlinear Galerkin method — 293
5.10 Inertial set — 319

**A Appendix — 341**
A.1 Basic notation and function space — 341
A.2 Sobolev embedding theorem and interpolation formula — 343
A.3 Fixed-point theorem — 346

**Bibliography — 349**

**Index — 361**