

Contents

Preface — V

| | |
|----------|--|
| 1 | An introduction to equations and optimization problems — 1 |
| 1.1 | Equations and their solutions — 1 |
| 1.2 | Origins and development of optimization problems — 2 |
| 1.3 | Structure of the book — 3 |
| 1.4 | Exercises — 4 |
| | |
| 2 | Solutions of algebraic equations — 7 |
| 2.1 | Solutions of polynomial equations — 7 |
| 2.1.1 | Polynomial equations of degrees 1 and 2 — 8 |
| 2.1.2 | Analytical solutions of cubic equations — 9 |
| 2.1.3 | Analytical solutions of quartic equation — 10 |
| 2.1.4 | Higher-degree equations and Abel–Ruffini theorem — 13 |
| 2.2 | Graphical methods for nonlinear equations — 13 |
| 2.2.1 | Smooth graphics for implicit functions — 13 |
| 2.2.2 | Univariate equations — 15 |
| 2.2.3 | Equations with two unknowns — 17 |
| 2.2.4 | Isolated equation solutions — 20 |
| 2.3 | Numerical solutions of algebraic equations — 20 |
| 2.3.1 | Newton–Raphson iterative algorithm — 20 |
| 2.3.2 | Direct solution methods with MATLAB — 25 |
| 2.3.3 | Accuracy specifications — 28 |
| 2.3.4 | Complex domain solutions — 29 |
| 2.4 | Accurate solutions of simultaneous equations — 31 |
| 2.4.1 | Analytical solutions of low-degree polynomial equations — 32 |
| 2.4.2 | Quasianalytical solutions of polynomial-type equations — 35 |
| 2.4.3 | Quasianalytical solutions of polynomial matrix equations — 37 |
| 2.4.4 | Quasianalytical solutions of nonlinear equations — 40 |
| 2.5 | Nonlinear matrix equations with multiple solutions — 41 |
| 2.5.1 | An equation solution idea and its implementation — 41 |
| 2.5.2 | Pseudopolynomial equations — 46 |
| 2.5.3 | A quasianalytical solver — 48 |
| 2.6 | Underdetermined algebraic equations — 49 |
| 2.7 | Exercises — 51 |
| | |
| 3 | Unconstrained optimization problems — 55 |
| 3.1 | Introduction to unconstrained optimization problems — 55 |
| 3.1.1 | The mathematical model of unconstrained optimization problems — 55 |
| 3.1.2 | Analytical solutions of unconstrained minimization problems — 56 |

| | | |
|----------|---|------------|
| 3.1.3 | Graphical solutions — | 56 |
| 3.1.4 | Local and global optimum solutions — | 58 |
| 3.1.5 | MATLAB implementation of optimization algorithms — | 60 |
| 3.2 | Direct solutions of unconstrained optimization problem with MATLAB — | 62 |
| 3.2.1 | Direct solution methods — | 62 |
| 3.2.2 | Control options in optimization — | 65 |
| 3.2.3 | Additional parameters — | 69 |
| 3.2.4 | Intermediate solution process — | 70 |
| 3.2.5 | Structured variable description of optimization problems — | 72 |
| 3.2.6 | Gradient information — | 73 |
| 3.2.7 | Optimization solutions from scattered data — | 77 |
| 3.2.8 | Parallel computation in optimization problems — | 78 |
| 3.3 | Towards global optimum solutions — | 79 |
| 3.4 | Optimization with decision variable bounds — | 83 |
| 3.4.1 | Univariate optimization problem — | 83 |
| 3.4.2 | Multivariate optimization problems — | 85 |
| 3.4.3 | Global optimum solutions — | 87 |
| 3.5 | Application examples of optimization problems — | 87 |
| 3.5.1 | Solutions of linear regression problems — | 88 |
| 3.5.2 | Least-squares curve fitting — | 89 |
| 3.5.3 | Shooting method in boundary value differential equations — | 93 |
| 3.5.4 | Converting algebraic equations into optimization problems — | 96 |
| 3.6 | Exercises — | 97 |
| 4 | Linear and quadratic programming — | 103 |
| 4.1 | An introduction to linear programming — | 104 |
| 4.1.1 | Mathematical model of linear programming problems — | 104 |
| 4.1.2 | Graphical solutions of linear programming problems — | 105 |
| 4.1.3 | Introduction to the simplex method — | 106 |
| 4.2 | Direct solutions of linear programming problems — | 110 |
| 4.2.1 | A linear programming problem solver — | 110 |
| 4.2.2 | Linear programming problems with multiple decision vectors — | 116 |
| 4.2.3 | Linear programming with double subscripts — | 117 |
| 4.2.4 | Transportation problem — | 118 |
| 4.3 | Problem-based description and solution of linear programming problems — | 122 |
| 4.3.1 | MPS file for linear programming problems — | 122 |
| 4.3.2 | Problem-based description of linear programming problems — | 124 |
| 4.3.3 | Conversions in linear programming problems — | 129 |
| 4.4 | Quadratic programming — | 130 |
| 4.4.1 | Mathematical quadratic programming models — | 131 |

| | | |
|----------|--|------------|
| 4.4.2 | Direct solutions of quadratic programming problems — | 131 |
| 4.4.3 | Problem-based quadratic programming problem description — | 132 |
| 4.4.4 | Quadratic programming problem with double subscripts — | 136 |
| 4.5 | Linear matrix inequalities — | 138 |
| 4.5.1 | Description of linear matrix inequality problems — | 138 |
| 4.5.2 | Lyapunov inequalities — | 139 |
| 4.5.3 | Classifications of LMI problems — | 141 |
| 4.5.4 | MATLAB solutions of LMI problems — | 142 |
| 4.5.5 | Optimization solutions with YALMIP toolbox — | 144 |
| 4.5.6 | Trials on nonconvex problems — | 146 |
| 4.5.7 | Problems with quadratic constraints — | 147 |
| 4.6 | Exercises — | 149 |
| 5 | Nonlinear programming — | 153 |
| 5.1 | Introduction to nonlinear programming — | 153 |
| 5.1.1 | Mathematical models of nonlinear programming problems — | 154 |
| 5.1.2 | Feasible regions and graphical methods — | 154 |
| 5.1.3 | Examples of numerical methods — | 157 |
| 5.2 | Direct solutions of nonlinear programming problems — | 159 |
| 5.2.1 | Direct solution using MATLAB — | 159 |
| 5.2.2 | Handling of earlier termination phenomenon — | 165 |
| 5.2.3 | Gradient information — | 166 |
| 5.2.4 | Solving problems with multiple decision vectors — | 168 |
| 5.2.5 | Complicated nonlinear programming problems — | 169 |
| 5.3 | Trials with global nonlinear programming solver — | 171 |
| 5.3.1 | Trials on global optimum solutions — | 171 |
| 5.3.2 | Nonconvex quadratic programming problems — | 174 |
| 5.3.3 | Concave-cost transportation problem — | 176 |
| 5.3.4 | Testing of the global optimum problem solver — | 178 |
| 5.3.5 | Handling piecewise objective functions — | 179 |
| 5.4 | Bilevel programming problems — | 181 |
| 5.4.1 | Bilevel linear programming problems — | 182 |
| 5.4.2 | Bilevel quadratic programming problem — | 183 |
| 5.4.3 | Bilevel program solutions with YALMIP Toolbox — | 184 |
| 5.5 | Nonlinear programming applications — | 185 |
| 5.5.1 | Maximum inner polygon inside a circle — | 185 |
| 5.5.2 | Semiinfinite programming problems — | 189 |
| 5.5.3 | Pooling and blending problem — | 193 |
| 5.5.4 | Optimization design of heat exchange network — | 196 |
| 5.5.5 | Solving nonlinear equations with optimization techniques — | 199 |
| 5.6 | Exercises — | 201 |

| | |
|----------|--|
| 6 | Mixed integer programming — 207 |
| 6.1 | Introduction to integer programming — 207 |
| 6.1.1 | Integer and mixed-integer programming problems — 207 |
| 6.1.2 | Computational complexity of integer programming problems — 208 |
| 6.2 | Enumeration methods for integer programming — 209 |
| 6.2.1 | An introduction to the enumeration method — 209 |
| 6.2.2 | Discrete programming — 213 |
| 6.2.3 | 0–1 programming — 214 |
| 6.2.4 | Trials on mixed-integer programming problems — 216 |
| 6.3 | Solutions of mixed-integer programming problems — 219 |
| 6.3.1 | Mixed-integer linear programming — 219 |
| 6.3.2 | Integer programming with YALMIP Toolbox — 222 |
| 6.3.3 | Mixed-integer nonlinear programming — 223 |
| 6.3.4 | A class of discrete programming problems — 226 |
| 6.3.5 | Solutions of ordinary discrete programming problems — 227 |
| 6.4 | Mixed 0–1 programming problems — 229 |
| 6.4.1 | 0–1 linear programming problems — 229 |
| 6.4.2 | 0–1 nonlinear programming problems — 233 |
| 6.5 | Mixed-integer programming applications — 235 |
| 6.5.1 | Optimal material usage — 235 |
| 6.5.2 | Assignment problem — 236 |
| 6.5.3 | Traveling salesman problem — 238 |
| 6.5.4 | Knapsack problems — 242 |
| 6.5.5 | Sudoku problems — 244 |
| 6.6 | Exercises — 247 |
| | |
| 7 | Multiobjective programming — 253 |
| 7.1 | Introduction to multiobjective programming — 253 |
| 7.1.1 | Background introduction — 253 |
| 7.1.2 | Mathematical model of multiobjective programming — 254 |
| 7.1.3 | Graphical solution of multiobjective programming problems — 255 |
| 7.2 | Multiobjective programming conversions and solutions — 257 |
| 7.2.1 | Least-squares solutions of multiobjective programming problems — 258 |
| 7.2.2 | Linear weighting conversions — 260 |
| 7.2.3 | Best compromise solution of linear programs — 261 |
| 7.2.4 | Least-squares linear programming — 263 |
| 7.3 | Pareto optimal solutions — 264 |
| 7.3.1 | Nonuniqueness of multiobjective programming — 265 |
| 7.3.2 | Dominant solutions and Pareto frontiers — 265 |
| 7.3.3 | Computations of Pareto frontier — 267 |
| 7.4 | Minimax problems — 268 |

7.5 Exercises — 275

8 Dynamic programming and shortest paths — 277

- 8.1 An introduction to dynamic programming — 277
 - 8.1.1 Concept and mathematical models in dynamic programming — 277
 - 8.1.2 Dynamic programming solutions of linear programming problems — 278
- 8.2 Shortest path problems in oriented graphs — 279
 - 8.2.1 Examples of oriented graphs — 280
 - 8.2.2 Manual solutions of shortest path problem — 281
 - 8.2.3 Solution with dynamic programming formulation — 282
 - 8.2.4 Matrix representation of graphs — 283
 - 8.2.5 Finding the shortest path — 284
 - 8.2.6 Dijkstra algorithm implementation — 288
- 8.3 Optimal paths for undigraphs — 290
 - 8.3.1 Matrix description — 290
 - 8.3.2 Route planning for cities with absolute coordinates — 292
- 8.4 Exercises — 293

9 Introduction to intelligent optimization methods — 297

- 9.1 Intelligent optimization algorithms — 297
 - 9.1.1 Genetic algorithms — 297
 - 9.1.2 Particle swarm optimization methods — 299
- 9.2 MATLAB Global Optimization Toolbox — 299
- 9.3 Examples and comparative studies of intelligent optimization methods — 301
 - 9.3.1 Unconstrained optimization problems — 302
 - 9.3.2 Constrained optimization problems — 305
 - 9.3.3 Mixed-integer programming — 312
 - 9.3.4 Discrete programming problems with the genetic algorithm — 315
- 9.4 Exercises — 317

Bibliography — 319

MATLAB function index — 321

Index — 325