## **Contents**

## Preface — V

1	Prerequisites for calculus — 1
1.1	Overview of calculus —— 1
1.2	Sets and numbers —— 6
1.2.1	Sets —— 6
1.2.2	Numbers —— 8
1.2.3	The least upper bound property —— 9
1.2.4	The extended real number system —— 11
1.2.5	Intervals —— 12
1.3	Functions —— 14
1.3.1	Definition of a function —— 14
1.3.2	Graph of a function —— 17
1.3.3	Some basic functions and their graphs —— 18
1.3.4	Building new functions —— 20
1.3.5	Fundamental elementary functions —— 31
1.3.6	Properties of functions —— 32
1.4	Exercises —— 37
2	Limits and continuity —— 41
2.1	Rates of change and derivatives —— 41
2.1 2.2	Limits of a function —— 42
	Limits of a function —— <b>42</b> Definition of a limit —— <b>42</b>
2.2	Limits of a function —— 42
2.2 2.2.1	Limits of a function —— <b>42</b> Definition of a limit —— <b>42</b>
2.2 2.2.1 2.2.2	Limits of a function —— 42  Definition of a limit —— 42  Properties of limits of functions —— 49
2.2 2.2.1 2.2.2 2.2.3	Limits of a function —— 42  Definition of a limit —— 42  Properties of limits of functions —— 49  Limit laws —— 50
2.2 2.2.1 2.2.2 2.2.3 2.2.4	Limits of a function —— 42  Definition of a limit —— 42  Properties of limits of functions —— 49  Limit laws —— 50  One-sided limits —— 55
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5	Limits of a function —— 42  Definition of a limit —— 42  Properties of limits of functions —— 49  Limit laws —— 50  One-sided limits —— 55  Limits involving infinity and asymptotes —— 59
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3	Limits of a function —— 42  Definition of a limit —— 42  Properties of limits of functions —— 49  Limit laws —— 50  One-sided limits —— 55  Limits involving infinity and asymptotes —— 59  Limits of sequences —— 68
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1	Limits of a function —— 42  Definition of a limit —— 42  Properties of limits of functions —— 49  Limit laws —— 50  One-sided limits —— 55  Limits involving infinity and asymptotes —— 59  Limits of sequences —— 68  Definitions and properties —— 68
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1 2.3.2	Limits of a function — 42 Definition of a limit — 42 Properties of limits of functions — 49 Limit laws — 50 One-sided limits — 55 Limits involving infinity and asymptotes — 59 Limits of sequences — 68 Definitions and properties — 68 Subsequences — 77
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1 2.3.2 2.4	Limits of a function — 42  Definition of a limit — 42  Properties of limits of functions — 49  Limit laws — 50  One-sided limits — 55  Limits involving infinity and asymptotes — 59  Limits of sequences — 68  Definitions and properties — 68  Subsequences — 77  Squeeze theorem and Cauchy's theorem — 78
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1 2.3.2 2.4 2.5	Limits of a function — 42  Definition of a limit — 42  Properties of limits of functions — 49  Limit laws — 50  One-sided limits — 55  Limits involving infinity and asymptotes — 59  Limits of sequences — 68  Definitions and properties — 68  Subsequences — 77  Squeeze theorem and Cauchy's theorem — 78  Infinitesimal functions and asymptotic functions — 86
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1 2.3.2 2.4 2.5 2.6	Limits of a function — 42  Definition of a limit — 42  Properties of limits of functions — 49  Limit laws — 50  One-sided limits — 55  Limits involving infinity and asymptotes — 59  Limits of sequences — 68  Definitions and properties — 68  Subsequences — 77  Squeeze theorem and Cauchy's theorem — 78  Infinitesimal functions and asymptotic functions — 86  Continuous and discontinuous functions — 91
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1 2.3.2 2.4 2.5 2.6 2.6.1	Limits of a function — 42  Definition of a limit — 42  Properties of limits of functions — 49  Limit laws — 50  One-sided limits — 55  Limits involving infinity and asymptotes — 59  Limits of sequences — 68  Definitions and properties — 68  Subsequences — 77  Squeeze theorem and Cauchy's theorem — 78  Infinitesimal functions and asymptotic functions — 86  Continuous and discontinuous functions — 91  Continuity and discontinuity — 91
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1 2.3.2 2.4 2.5 2.6 2.6.1 2.6.2	Limits of a function — 42  Definition of a limit — 42  Properties of limits of functions — 49  Limit laws — 50  One-sided limits — 55  Limits involving infinity and asymptotes — 59  Limits of sequences — 68  Definitions and properties — 68  Subsequences — 77  Squeeze theorem and Cauchy's theorem — 78  Infinitesimal functions and asymptotic functions — 86  Continuous and discontinuous functions — 91  Continuous functions — 94
2.2 2.2.1 2.2.2 2.2.3 2.2.4 2.2.5 2.3 2.3.1 2.3.2 2.4 2.5 2.6 2.6.1 2.6.2 2.6.3	Limits of a function — 42  Definition of a limit — 42  Properties of limits of functions — 49  Limit laws — 50  One-sided limits — 55  Limits involving infinity and asymptotes — 59  Limits of sequences — 68  Definitions and properties — 68  Subsequences — 77  Squeeze theorem and Cauchy's theorem — 78  Infinitesimal functions and asymptotic functions — 86  Continuous and discontinuous functions — 91  Continuous functions — 94  Theorems on continuous functions — 99



3	The derivative —— 121
3.1	Derivative of a function at a point —— 121
3.1.1	Instantaneous rates of change and derivatives revisited —— 121
3.1.2	One-sided derivatives —— 128
3.1.3	A function may fail to have a derivative at a point —— 129
3.2	Derivative as a function —— 133
3.2.1	Graphing the derivative of a function —— 134
3.2.2	Derivatives of some basic functions —— 135
3.3	Derivative laws —— 139
3.4	Derivative of an inverse function —— 143
3.5	Differentiating a composite function – the chain rule —— 147
3.6	Derivatives of higher orders —— 152
3.7	Implicit differentiation —— 154
3.8	Functions defined by parametric and polar equations —— 159
3.8.1	Functions defined by parametric equations —— 159
3.8.2	Polar curves —— 163
3.9	Related rates of change —— 166
3.10	The tangent line approximation and the differential —— 167
3.10.1	Linearization —— 167
3.10.2	Differentials —— 170
3.11	Derivative rules – summary —— 174
3.12	Exercises —— 175
4	Applications of the derivative —— 181
4.1	Extreme values and the candidate theorem —— 181
4.2	The mean value theorem —— 188
4.3	Monotonic functions and the first derivative test —— 196
4.3.1	Monotonic functions —— 196
4.3.2	The first derivative test —— 199
4.4	Extended mean value theorem and the L'Hôpital rules —— 201
4.4.1	Extended mean value theorem —— 201
4.4.2	The indeterminate forms $\frac{0}{0}$ , $\infty - \infty$ , $\frac{\infty}{\infty}$ , and $0 \times \infty$ — 203
4.5	Taylor's theorem —— 209
4.5.1	The error analysis for the linear approximation —— 209
4.5.2	The quadratic approximation —— 210
4.5.3	Taylor's theorem —— 214
4.6	Concave functions and the second derivative test —— 219
4.6.1	Concave functions —— 219
4.6.2	The second derivative test —— 225
4.7	Extreme values of functions revisited —— 227
4.8	Curve sketching —— 231

4.9.1	Decimal search —— 234
4.9.2	Newton's method —— 236
4.10	Curvatures and the differential of the arc length —— 238
4.11	Exercises —— 243
5	The definite integral —— 249
5.1	Definite integrals and properties — 249
5.1.1	Introduction —— 249
5.1.2	Properties of the definite integral —— 259
5.1.3	Interpreting $\int_a^b f(x) dx$ in terms of area —— 265
5.1.4	Interpreting $\int_a^b v(t) dt$ as a distance or displacement — 267
5.2	The fundamental theorem of calculus —— <b>267</b>
5.3	Numerical integration —— 275
5.3.1	Trapezoidal rule —— 276
5.3.2	Simpson's rule —— 277
5.4	Exercises —— 279
6	Techniques for integration and improper integrals —— 285
6.1	Indefinite integrals —— 285
6.1.1	Definition of indefinite integrals and basic antiderivatives — 285
6.1.2	Differential equations —— 289
6.1.3	Substitution in indefinite integrals —— 293
6.1.4	Further results using integration by substitution —— 297
6.1.5	Integration by parts —— 300
6.1.6	Partial fractions in integration —— 304
6.1.7	Rationalizing substitutions —— 312
6.2	Substitution in definite integrals —— 313
6.3	Integration by parts in definite integrals —— 317
6.4	Improper integrals —— 318
6.4.1	Improper integrals of the first kind —— 318
6.4.2	Improper integrals of the second kind —— 322
6.5	Exercises —— 326
7	Applications of the definite integral —— 333
7.1	Areas, volumes, and arc lengths —— 333
7.1.1	The area of the region between two curves —— 333
7.1.2	Volumes of solids —— 337
7.1.3	Arc length —— 339
7.2	Applications in other disciplines —— 344
7.2.1	Displacement and distance —— 344
7.2.2	Work done by a force —— 345

## X — Contents

7.2.3	Fluid pressure —— 346
7.2.4	Center of mass —— 347
7.2.5	Probability —— 349
7.3	Exercises —— 350
8	Infinite series, sequences, and approximations —— 355
8.1	Infinite sequences —— 355
8.2	Infinite series —— 357
8.2.1	Definition of infinite series —— 357
8.2.2	Properties of convergent series —— 359
8.3	Tests for convergence —— 363
8.3.1	Series with nonnegative terms —— 363
8.3.2	Series with negative and positive terms —— 371
8.4	Power series and Taylor series —— 375
8.4.1	Power series —— 375
8.4.2	Working with power series —— 381
8.4.3	Taylor series —— 383
8.4.4	Applications of power series —— 391
8.5	Fourier series —— <b>393</b>
8.5.1	Fourier series expansion with period $2\pi$ — 394
8.5.2	Fourier cosine and sine series with period $2\pi$ — 399
8.5.3	The Fourier series expansion with period 21 —— 400
8.5.4	Fourier series with complex terms —— 403
8.6	Exercises —— 404

Index —— 411