

Contents

Preface — V

1	Introduction to linear algebra and matrix computation — 1
1.1	Matrices and linear equations — 1
1.1.1	Matrix representation of tables — 1
1.1.2	Modeling and solving linear equations — 3
1.2	Introduction of linear algebra — 8
1.2.1	Mathematical development of linear algebra — 8
1.2.2	Numerical linear algebra — 10
1.3	Outline of the book — 10
1.4	Exercises of the chapter — 12
2	Representation and fundamental computations of matrices — 13
2.1	Input of an ordinary matrix — 13
2.2	Input of special matrices — 15
2.2.1	Matrices of zeros, ones, and identity matrices — 15
2.2.2	Matrices with random elements — 16
2.2.3	Hankel matrix — 18
2.2.4	Diagonal matrices — 19
2.2.5	Hilbert matrix and its inverse — 22
2.2.6	Companion matrices — 22
2.2.7	Vandermonde matrices — 23
2.2.8	Some commonly used test matrices — 24
2.3	Symbolic matrix input — 26
2.3.1	Input of special symbolic matrices — 26
2.3.2	Generating arbitrary constant matrices — 27
2.3.3	Arbitrary matrix function input — 28
2.4	Input of sparse matrices — 29
2.5	Fundamental matrix computations — 32
2.5.1	Handling complex matrices — 32
2.5.2	Transposition and rotation of matrices — 33
2.5.3	Algebraic computation of matrices — 35
2.5.4	Kronecker products and sums — 39
2.6	Calculus computations with matrices — 40
2.6.1	Matrix derivatives — 41
2.6.2	Integrals of matrix functions — 42
2.6.3	Jacobian matrices of vector functions — 43
2.6.4	Hessian matrices — 43
2.7	Exercises — 44

3	Fundamental analysis of matrices — 47
3.1	Determinants — 47
3.1.1	Determinant definition and properties — 47
3.1.2	Determinants of small-scale matrices — 48
3.1.3	MATLAB solution of determinant problems — 52
3.1.4	Determinants of special matrices of any sizes — 55
3.1.5	Cramer's rule in linear equations — 57
3.1.6	Positive and totally positive matrices — 59
3.2	Simple analysis of matrices — 60
3.2.1	The traces — 60
3.2.2	Linear independence and matrix rank — 61
3.2.3	Norms — 63
3.2.4	Vector space — 65
3.3	Inverse and generalized inverse matrices — 66
3.3.1	Inverse matrices — 66
3.3.2	Derivatives of inverse matrix — 68
3.3.3	MATLAB based inverse matrix evaluation — 69
3.3.4	Reduced row echelon form — 72
3.3.5	Generalized inverse — 74
3.4	Characteristic polynomials and eigenvalues — 77
3.4.1	Characteristic polynomials — 77
3.4.2	Finding the roots of polynomial equations — 79
3.4.3	Eigenvalues and eigenvectors — 80
3.4.4	Generalized eigenvalues and eigenvectors — 84
3.4.5	Gershgorin discs and diagonally dominant matrices — 86
3.5	Matrix polynomials — 87
3.5.1	Solutions of matrix polynomial problems — 88
3.5.2	Minimal polynomials of matrices — 89
3.5.3	Conversions between symbolic and numerical polynomials — 90
3.6	Problems — 92
4	Fundamental transformation and decomposition of matrices — 95
4.1	Similarity transform and orthogonal matrices — 95
4.1.1	Similarity transform — 95
4.1.2	Orthogonal matrices and orthonormal bases — 96
4.2	Elementary row transforms — 98
4.2.1	Three types of elementary row transforms — 98
4.2.2	Inverse through elementary row transforms — 100
4.2.3	Computing inverses with pivot methods — 102
4.3	Triangular decomposition of matrices — 103
4.3.1	Gaussian elimination of linear equations — 103
4.3.2	Triangular factorization and implementation — 105

4.3.3	MATLAB triangular factorization solver — 106
4.4	Cholesky factorization — 108
4.4.1	Cholesky factorization of symmetric matrices — 108
4.4.2	Quadratic forms of symmetric matrices — 109
4.4.3	Positive definite and regular matrices — 110
4.4.4	Cholesky factorization of nonpositive definite matrices — 112
4.5	Companion and Jordan transforms — 113
4.5.1	Companion form transformation — 113
4.5.2	Matrix diagonalization — 114
4.5.3	Jordan transform — 115
4.5.4	Real Jordan transforms for matrices with repeated complex eigenvalues — 117
4.5.5	Simultaneous diagonalization — 119
4.6	Singular value decomposition — 120
4.6.1	Singular values and condition numbers — 121
4.6.2	Singular value decomposition of rectangular matrices — 123
4.6.3	SVD based simultaneous diagonalization — 124
4.7	Givens and Householder transforms — 125
4.7.1	Two-dimensional coordinate rotation transform — 125
4.7.2	Givens transform of ordinary matrices — 128
4.7.3	Householder transform — 130
4.8	Problems — 131
5	Solutions of matrix equations — 135
5.1	Linear equations — 135
5.1.1	Unique solutions — 136
5.1.2	Linear equations with infinitely many solutions — 140
5.1.3	Inconsistent systems of equations — 143
5.1.4	Graphical interpretation of linear solutions — 144
5.2	Other forms of simple linear equations — 146
5.2.1	$\mathbf{XA} = \mathbf{B}$ equations — 146
5.2.2	$\mathbf{AXB} = \mathbf{C}$ equations — 148
5.2.3	Kronecker product-based solutions — 150
5.2.4	Multiterm $\mathbf{AXB} = \mathbf{C}$ equations — 151
5.3	Lyapunov equations — 152
5.3.1	Continuous Lyapunov equations — 152
5.3.2	2×2 Lyapunov equations — 154
5.3.3	Analytical solution of Lyapunov equations — 155
5.3.4	Stein equations — 156
5.3.5	Discrete Lyapunov equation — 157
5.4	Sylvester equations — 158
5.4.1	Numerical solutions of Sylvester equations — 158

5.4.2	Analytical solution of Sylvester equation —	159
5.4.3	Sylvester equations with constant parameters —	161
5.4.4	Multiterm Sylvester equations —	162
5.5	Nonlinear matrix equations —	163
5.5.1	Riccati equations —	163
5.5.2	Solutions of general nonlinear equations —	165
5.5.3	Variations of Riccati equations —	169
5.5.4	Numerical solutions of nonlinear matrix equations —	170
5.6	Diophantine equations —	171
5.6.1	Coprime-ness of polynomials —	172
5.6.2	Diophantine polynomial equations —	172
5.6.3	Solutions of pseudo-polynomial equations —	175
5.7	Problems —	176
6	Matrix functions —	181
6.1	Nonlinear computation of matrix elements —	182
6.1.1	Integer rounding and rationalization —	182
6.1.2	Transcendental function computations —	183
6.1.3	Vector sorting, minimization, and maximization —	187
6.1.4	Means, variances, and standard deviations —	188
6.2	Matrix exponentials —	189
6.2.1	Definitions and properties of matrix functions —	189
6.2.2	Computing matrix exponentials —	190
6.2.3	Taylor series-based truncation algorithm —	190
6.2.4	A Cayley–Hamilton theorem-based algorithm —	192
6.2.5	Direct MATLAB solver —	193
6.2.6	Jordan transform-based approach —	195
6.3	Matrix logarithmic and square root functions —	196
6.3.1	Matrix logarithmic function —	196
6.3.2	Square root of a matrix —	198
6.4	Matrix trigonometric functions —	199
6.4.1	Computation of matrix trigonometric functions —	199
6.4.2	Power series-based trigonometric function evaluation —	200
6.4.3	Analytical evaluations of matrix trigonometric functions —	202
6.5	Arbitrary matrix functions —	204
6.5.1	Nilpotent matrices —	204
6.5.2	Jordan transform-based matrix function evaluations —	205
6.5.3	User-defined matrix functions —	208
6.6	Matrix powers —	210
6.6.1	Matrix powers via Jordan transform —	210
6.6.2	A matrix power solver —	211
6.6.3	Matrix powers via the z transform —	213

6.6.4	Computing k^A — 214
6.7	Problems — 215
7	Linear algebra applications — 217
7.1	Linear equations applications — 217
7.1.1	Electric network analysis — 217
7.1.2	Analysis of truss structures — 224
7.1.3	Balancing chemical reaction equations — 224
7.2	Applications in linear control systems — 227
7.2.1	Control system model conversions — 227
7.2.2	Qualitative analysis of linear systems — 229
7.2.3	Transmission zeros in multivariate systems — 231
7.2.4	Direct solutions of linear differential equations — 232
7.3	Introduction to digital image processing — 233
7.3.1	Image input and display — 234
7.3.2	Singular value decomposition applications — 235
7.3.3	Resizing and rotating images — 237
7.3.4	Image enhancement — 239
7.4	Graph theory and applications — 241
7.4.1	Oriented graph descriptions — 242
7.4.2	Dijkstra algorithm and applications — 244
7.4.3	Simplification of control system block diagrams — 247
7.5	Solving difference equations — 251
7.5.1	Analytical solutions — 252
7.5.2	Numerical solutions of linear time-varying difference equations — 254
7.5.3	Linear time-invariant difference equations — 255
7.5.4	Numerical solutions of nonlinear difference equations — 257
7.5.5	Markov chain simulation — 258
7.6	Data fitting and analysis — 260
7.6.1	Linear regression — 261
7.6.2	Polynomial fitting — 264
7.6.3	Chebyshev polynomials — 265
7.6.4	Bézier curves — 267
7.6.5	Principal component analysis — 269
7.7	Exercises of the chapter — 272

Bibliography — 279

MATLAB function index — 281

Index — 285