

Contents

1 Conformal Einstein Evolution

<i>Helmut Friedrich</i>	1
1.1 Introduction	1
1.2 The Conformal Field Equations	4
1.2.1 Conformal Geometry	6
1.2.2 Derivation of the Conformal Field Equations	9
1.3 The Penrose Proposal	21
1.4 Asymptotic Behaviour of Vacuum Fields with Vanishing Cosmological Constant	29
1.4.1 The Hyperboloidal Initial Value Problem	30
1.4.2 On the Existence of Asymptotically Simple Vacuum Solutions	31
1.4.3 The Regular Finite Cauchy Problem	33
1.4.4 Time-Like Infinity	42
References	46

2 Some Global Results for Asymptotically Simple Space-Times

<i>Gregory J. Galloway</i>	51
2.1 Introduction	51
2.2 The Null Splitting Theorem	53
2.3 Proof of Theorem 2.1	55
2.4 Concluding Remarks	58
References	59

3 Black Holes

<i>Piotr T. Chruściel</i>	61
3.1 Introduction	61
3.2 Experimental Evidence	62
3.3 Causality for Symmetric Hyperbolic Systems	65
3.3.1 Dumb Holes	69
3.3.2 Optical Holes	70
3.3.3 Trapped Surfaces	70
3.4 Standard Black Holes	71
3.4.1 Scri Regularity Conditions, and the Area Theorem	75
3.5 Horizons	77
3.6 Apparent Horizons	79

3.7	Classification of Stationary Solutions (“No Hair Theorems”)	80
3.8	Black Holes Without Scri	83
3.8.1	Naive Black Holes	84
3.8.2	Quasi-local Black Holes	86
3.8.3	Finding Horizons	90
	References	96
4	Conformal Geometry, Differential Equations and Associated Transformations	
	<i>Simonetta Frittelli, Niky Kamran, Ezra T. Newman</i>	103
4.1	Introduction	103
4.2	Example of Contact-Envelope Transformation	105
4.3	Generalizations	109
4.3.1	Three-Dimensional Conformal Lorentzian Geometries	109
4.3.2	Four-Dimensional Conformal Lorentzian Geometries	110
	References	111
5	Twistor Geometry of Conformal Infinity	
	<i>Roger Penrose</i>	113
5.1	Non-linear Gravitons	113
5.2	The Reasonableness of $\mathcal{S}^+$	115
5.3	The Construction of Projective Twistor Space $\mathbb{PT}$ from $\mathcal{S}^+$	115
5.4	The Construction of the Full Twistor Space $\mathcal{T}$ from $\mathcal{S}^+$	117
5.5	The Local Structure of Twistor Space $\mathbb{PT}$	118
5.6	Present Status of the Role of $\mathcal{T}$ in Encoding Ricci-Flatness	119
	References	120
6	Isotropic Cosmological Singularities	
	<i>K. Paul Tod</i>	123
6.1	Introduction	123
6.2	Formalism and Extensions	125
6.3	Review of Polytropic Perfect Fluid Case	128
6.4	Further Matter Models	131
6.4.1	Massive Einstein-Vlasov	132
6.4.2	Scalar Fields	132
6.4.3	Einstein-Yang-Mills-Vlasov	132
6.4.4	Einstein-Boltzmann	132
6.5	Conclusion and Future Possibilities	133
	References	133
7	Polyhomogeneous Expansions Close to Null and Spatial Infinity	
	<i>Juan Antonio Valiente Kroon</i>	135
7.1	Introduction	135
7.2	Minkowski Space-Time Close to Null and Spatial Infinity	136
7.3	Linearised Gravity in the F-Gauge	138

7.3.1	Initial Data for Linearised Gravity	140
7.4	A Regularity Condition at Spatial Infinity	142
7.5	Polyhomogeneous Expansions	145
7.5.1	A Subtraction Argument	145
7.5.2	An Investigation of Expansions Close to Null Infinity	147
7.5.3	Concluding Remarks	157
	References	158
8	Asymptotically Flat and Regular Cauchy Data	
<i>Sergio Dain</i>		161
8.1	Introduction	161
8.2	Solution of the Hamiltonian Constraint with Logarithmic Terms	172
8.3	Explicit Solutions of the Momentum Constraint	173
8.3.1	The Momentum Constraint on Euclidean Space	173
8.3.2	Axially Symmetric Initial Data	176
8.4	Main Ideas in the Proof of Theorem 8.1	178
8.5	Final Comments.....	180
	References	180
9	Construction of Hyperboloidal Initial Data	
<i>Lars Andersson</i>		183
9.1	Introduction	183
9.2	Preliminaries.....	184
9.3	Conformal Rescalings of Minkowski Space	185
9.4	Conformal Constraint Equations.....	187
9.4.1	Constant Mean Curvature Hypersurfaces.....	188
9.4.2	Degenerate Elliptic Equations	189
9.4.3	Regularity of Solutions to the Conformal Constraint Equations	190
9.5	The Initial Value Problem	191
9.5.1	Gauge Condition at ∂M	191
9.5.2	Evolution at ∂M	192
9.6	Discussion	193
	References	193
10	Exploring the Conformal Constraint Equations	
<i>Adrian Butscher</i>		195
10.1	Introduction	195
10.2	The Conformal Constraint Equations	197
10.2.1	Deriving the Equations	197
10.2.2	Reduction to the Extended Constraint Equations	201
10.2.3	Properties of the Extended Constraint Equations	202
10.3	Asymptotically Flat Solutions of the Extended Constraint Equations in the Time Symmetric Case.....	206
10.3.1	Statement of the Main Theorem	206

10.3.2	Formulating an Elliptic Problem	208
10.3.3	Choosing the Banach Spaces	209
10.3.4	First Attempt to Solve the Associated System	212
10.3.5	Reestablishing Surjectivity and Solving the Associated System	215
10.3.6	Satisfying the Harmonic Coordinate Condition	217
	References	221
11	Criteria for (In)finite Extent of Static Perfect Fluids	
	<i>Walter Simon</i>	223
11.1	Introduction	223
11.2	The Main Theorem	225
11.3	The Virial Theorem	230
11.4	Proof of the Main Theorem	232
11.5	Discussion	235
	References	237
12	Problems and Successes in the Numerical Approach to the Conformal Field Equations	
	<i>Sascha Husa</i>	239
12.1	Introduction	239
12.2	Algorithms	240
12.2.1	Problem Overview	240
12.2.2	Construction of “Extended” Hyperboloidal Initial Data ..	243
12.2.3	Black Hole Initial Data	246
12.2.4	Numerical Setup for Evolutions	247
12.2.5	Physics Extraction	248
12.3	Results for Weak Data	248
12.4	Computational Aspects	251
12.5	Discussion	255
	References	257
13	Some Aspects of the Numerical Treatment of the Conformal Field Equations	
	<i>Jörg Frauendiener</i>	261
13.1	Introduction	261
13.2	The Andersson-Chruściel-Friedrich Procedure	263
13.3	The Lichnérowicz–Yamabe–Equation	266
13.4	Constructing Initial Data	273
13.5	Conclusion	278
	References	280
14	Data for the Numerical Calculation of the Kruskal Space-Time	
	<i>Bernd G. Schmidt</i>	283
14.1	Introduction	283
14.2	Conformal Extension of the Kruskal Space-Time	284

14.3	A Space-Like Hypersurface	
	Intersecting $\mathcal{S}_L^+$ and $\mathcal{S}_R^+$	289
14.4	A Foliation Intersecting Both $\mathcal{S}$	292
14.5	Numerical Calculation of the Kruskal Space-Time	293
	References	295
15	Numerics of the Characteristic Formulation	
	in Bondi Variables. Where We Are and What Lies Ahead	
<i>Luis Lehner</i>		297
15.1	Introduction	297
15.2	Characteristic Formulation of GR in Bondi Variables	298
	15.2.1 Initial Boundary Value Problem	299
	15.2.2 News	301
	15.2.3 Inertial Coordinates	302
15.3	Numerical Details	304
15.4	Applications	305
	15.4.1 Black Hole-Star Binaries	306
	15.4.2 Binary Black Hole Problem	308
15.5	Final Comments	310
	References	311
16	Numerical Experiments at Null Infinity	
<i>Robert A. Bartnik, Andrew H. Norton</i>		313
16.1	Introduction	313
16.2	NQS Metrics	315
16.3	NQS Formal Asymptotics	317
16.4	Genericity	321
16.5	The NQS Code	322
16.6	Numerical Results	323
	References	325
17	Local Characteristic Algorithms	
	for Relativistic Hydrodynamics	
<i>José A. Font</i>		327
17.1	Introduction	327
17.2	Relativistic Hydrodynamic Equations	329
17.3	High-Resolution Numerical Schemes	332
17.4	Applications	336
	17.4.1 Shock Tube Test	336
	17.4.2 Gravitational Collapse of Supermassive Stars	338
	17.4.3 Null Cone Evolution of Relativistic Stars	340
17.5	Summary	343
	References	344

18 Simulations of Generic Singularities in Harmonic Coordinates

<i>David Garfinkle</i>	349
18.1 Introduction	349
18.2 Equations and Numerical Methods	351
18.3 Results	352
18.4 Discussion	356
References	357

19 Some Mathematical and Numerical Questions Connected with First and Second Order Time-Dependent Systems of Partial Differential Equations

<i>Heinz-O. Kreiss, Omar E. Ortiz</i>	359
19.1 Introduction	359
19.2 Well Posed Problems	360
19.2.1 First Order Systems	360
19.2.2 Second Order Systems	362
19.3 Second Order Initial Value Formulations for General Relativity ..	364
19.4 Difference Approximations	366
19.5 Constraints	368
References	370