

Giovanni Gallavotti

Statistical Mechanics

A Short Treatise

With 21 Figures



Springer

Table of Contents

| | |
|---|-----------|
| 1. Classical Statistical Mechanics | 1 |
| 1.1 Introduction | 1 |
| 1.2 Microscopic Dynamics | 3 |
| 1.3 Time Averages and the Ergodic Hypothesis | 10 |
| 1.4 Recurrence Times and Macroscopic Observables | 14 |
| 1.5 Statistical Ensembles or "Monodes" and Models of Thermodynamics. Thermodynamics Without Dynamics | 17 |
| 1.6 Models of Thermodynamics. Microcanonical and Canonical Ensembles and the Ergodic Hypothesis | 20 |
| 1.7 Critique of the Ergodic Hypothesis | 24 |
| 1.8 Approach to Equilibrium and Boltzmann's Equation. Ergodicity and Irreversibility | 27 |
| 1.9 A Historical Note. The Etymology of the Word "Ergodic" and the Heat Theorems | 36 |
| Appendix 1.A1 Monocyclic Systems, Keplerian Motions and Ergodic Hypothesis | 44 |
| Appendix 1.A2 Grad-Boltzmann Limit and Lorentz's Gas | 48 |
| 2. Statistical Ensembles | 57 |
| 2.1 Statistical Ensembles as Models of Thermodynamics | 57 |
| 2.2 Canonical and Microcanonical Ensembles: Orthodicity | 60 |
| 2.3 Equivalence Between Canonical and Microcanonical Ensembles | 68 |
| 2.4 Non-equivalence of the Canonical and Microcanonical Ensembles. Phase Transitions. Boltzmann's Constant | 73 |
| 2.5 The Grand Canonical Ensemble and Other Orthodic Ensembles | 76 |
| 2.6 Some Technical Aspects | 84 |
| 3. Equipartition and Critique | 89 |
| 3.1 Equipartition and Other Paradoxes and Applications of Statistical Mechanics | 89 |
| 3.2 Classical Statistical Mechanics When Cell Sizes Are Not Negligible | 95 |

| | | |
|-----------|--|------------|
| 3.3 | Introduction to Quantum Statistical Mechanics | 103 |
| 3.4 | Philosophical Outlook on the Foundations of Statistical Mechanics | 107 |
| 4. | Thermodynamic Limit and Stability | 111 |
| 4.1 | The Meaning of the Stability Conditions | 111 |
| 4.2 | Stability Criteria | 114 |
| 4.3 | Thermodynamic Limit | 117 |
| 5. | Phase Transitions | 129 |
| 5.1 | Virial Theorem, Virial Series and van der Waals Equation . | 129 |
| 5.2 | The Modern Interpretation of van der Waals' Approximation | 136 |
| 5.3 | Why a Thermodynamic Formalism? | 142 |
| 5.4 | Phase Space in Infinite Volume and Probability Distributions on It. Gibbs Distributions ... | 144 |
| 5.5 | Variational Characterization of Translation Invariant Gibbs Distributions | 147 |
| 5.6 | Other Characterizations of Gibbs Distributions. The DLR Equations | 153 |
| 5.7 | Gibbs Distributions and Stochastic Processes | 155 |
| 5.8 | Absence of Phase Transitions: $d = 1$. Symmetries: $d = 2$... | 157 |
| 5.9 | Absence of Phase Transitions: High Temperature and the KS Equations | 161 |
| 5.10 | Phase Transitions and Models | 167 |
| | Appendix 5.A1 Absence of Phase Transition in Non-Nearest-Neighbor One-Dimensional Systems | 172 |
| 6. | Coexistence of Phases | 175 |
| 6.1 | The Ising Model. Inequivalence of Canonical and Grand Canonical Ensembles | 175 |
| 6.2 | The Model. Grand Canonical and Canonical Ensembles. Their Inequivalence | 176 |
| 6.3 | Boundary Conditions. Equilibrium States | 178 |
| 6.4 | The Ising Model in One and Two Dimensions and Zero Field | 180 |
| 6.5 | Phase Transitions. Definitions | 182 |
| 6.6 | Geometric Description of the Spin Configurations | 184 |
| 6.7 | Phase Transitions. Existence | 188 |
| 6.8 | Microscopic Description of the Pure Phases | 189 |
| 6.9 | Results on Phase Transitions in a Wider Range of Temperature | 192 |
| 6.10 | Separation and Coexistence of Pure Phases. Phenomenological Considerations | 196 |

| | |
|--|------------|
| 6.11 Separation and Coexistence of Phases. Results | 198 |
| 6.12 Surface Tension in Two Dimensions. Alternative Description of the Separation Phenomena | 199 |
| 6.13 The Structure of the Line of Separation. What a Straight Line Really Is | 200 |
| 6.14 Phase Separation Phenomena and Boundary Conditions. Further Results | 202 |
| 6.15 Further Results, Some Comments and Some Open Problems | 205 |
| 7. Exactly Soluble Models | 209 |
| 7.1 Transfer Matrix in the Ising Model: Results in $d = 1, 2$ | 209 |
| 7.2 Meaning of Exact Solubility and the Two-Dimensional Ising Model | 211 |
| 7.3 Vertex Models | 214 |
| 7.4 A Nontrivial Example of Exact Solution: The Two-Dimensional Ising Model | 221 |
| 7.5 The Six-Vertex Model and Bethe's Ansatz | 227 |
| 8. Brownian Motion | 233 |
| 8.1 Brownian Motion and Einstein's Theory | 233 |
| 8.2 Smoluchowski's Theory | 239 |
| 8.3 The Uhlenbeck-Ornstein Theory | 242 |
| 8.4 Wiener's Theory | 246 |
| 9. Coarse Graining and Nonequilibrium | 253 |
| 9.1 Ergodic Hypothesis Revisited | 253 |
| 9.2 Timed Observations and Discrete Time | 257 |
| 9.3 Chaotic Hypothesis. Anosov Systems | 260 |
| 9.4 Kinematics of Chaotic Motions. Anosov Systems | 265 |
| 9.5 Symbolic Dynamics and Chaos | 270 |
| 9.6 Statistics of Chaotic Attractors. SRB Distributions | 278 |
| 9.7 Entropy Generation. Time Reversibility and Fluctuation Theorem. Experimental Tests of the Chaotic Hypothesis | 281 |
| 9.8 Fluctuation Patterns | 287 |
| 9.9 "Conditional Reversibility" and "Fluctuation Theorems" .. | 288 |
| 9.10 Onsager Reciprocity and Green-Kubo's Formula | 292 |
| 9.11 Reversible Versus Irreversible Dissipation. Nonequilibrium Ensembles? | 294 |
| Appendix 9.A1 Mécanique statistique hors équilibre: l'héritage de Boltzmann | 298 |
| Appendix 9.A2 Heuristic Derivation of the SRB Distribution ... | 308 |

XIV Table of Contents

| | |
|--|-----|
| Appendix 9.A3 Aperiodic Motions Can Be Regarded as Periodic with Infinite Period! | 310 |
| Appendix 9.A4 Gauss' Least Constraint Principle | 311 |
| Bibliography | 313 |
| Name Index | 331 |
| Subject Index | 333 |