

Contents

Preface — VII

1 Basic definitions — 1

- 1.1 Canonical systems — 1
- 1.2 Singular intervals — 5
- 1.3 Transformations — 6
- 1.4 The Hilbert space L_H^2 — 9
- 1.5 Notes — 10

2 Symmetric and self-adjoint relations — 11

- 2.1 The minimal and maximal relations of a canonical system — 11
- 2.2 Von Neumann theory of symmetric relations — 19
- 2.3 Self-adjoint realizations of canonical systems — 24
- 2.4 The multi-valued part — 28
- 2.5 More on deficiency indices — 32
- 2.6 Notes — 34

3 Spectral representation — 35

- 3.1 Spectral representation of relations — 35
- 3.2 The resolvent as an integral operator — 36
- 3.3 A limit point/limit circle criterion — 39
- 3.4 Weyl theory — 40
- 3.5 Spectral representation of canonical systems — 49
- 3.6 General boundary conditions — 56
- 3.7 Two limit point endpoints — 58
- 3.8 Notes — 67

4 Transfer matrices and de Branges spaces — 69

- 4.1 Review of complex analysis — 69
- 4.2 De Branges spaces — 72
- 4.3 Spectral representation and de Branges spaces — 76
- 4.4 Transfer matrices — 84
- 4.5 Regular de Branges spaces — 93
- 4.6 The type of a transfer matrix — 95
- 4.7 Transfer matrices and singular intervals — 97
- 4.8 Notes — 103

5 Inverse spectral theory — 105

- 5.1 Introduction — **105**
- 5.2 Metrics on canonical systems and spectral data — **106**
- 5.3 Existence of a canonical system with given m function — **112**
- 5.4 The ordering theorem for de Branges spaces — **116**
- 5.5 Existence of a canonical system with given transfer matrix — **128**
- 5.6 Uniqueness — **131**
- 5.7 Notes — **135**

6 Some applications — 139

- 6.1 Generalized and classical moment problems and the Nevanlinna parametrization — **139**
- 6.2 Existence of moments — **145**
- 6.3 Diagonal canonical systems — **148**
- 6.4 Dirac systems — **151**
- 6.5 Notes — **157**

7 The absolutely continuous spectrum — 159

- 7.1 Twisted shifts and Toda maps — **159**
- 7.2 Convergence of Herglotz functions — **163**
- 7.3 Reflectionless canonical systems — **171**
- 7.4 Semi-continuity of R and Σ_{ac} — **174**
- 7.5 Reflectionless limit points — **178**
- 7.6 Notes — **187**

Bibliography — 189

Index — 193