

Contents

Preface — VII

1 Basic Topology — 1

- 1.1 Basic Notions — 1
- 1.2 Separation and Countability Properties – Convergence — 9
- 1.3 Weak, Product, and Quotient Topologies — 19
- 1.4 Connectedness and Compactness — 25
- 1.5 Metric Spaces – Baire Category — 44
- 1.6 Function Spaces — 59
- 1.7 Semicontinuous Functions – Miscellaneous Notions — 65
- 1.8 Remarks — 73

2 Measure Theory — 83

- 2.1 Basic Notions, Measures, and Outer Measures — 83
- 2.2 Measurable Functions – Integration — 98
- 2.3 Convergence Theorems and L^p -Spaces — 110
- 2.4 Signed Measures and Radon–Nikodym Theorem — 127
- 2.5 Regular and Radon Measures — 137
- 2.6 Analytic (Souslin) Sets — 147
- 2.7 Selection and Projection Theorems — 156
- 2.8 Remarks — 167

3 Basic Functional Analysis — 179

- 3.1 Topological Vector Spaces, Hahn–Banach Theorem — 179
- 3.2 Three Fundamental Theorems — 197
- 3.3 Weak and Weak* Topologies — 207
- 3.4 Separable and Reflexive Banach Spaces — 216
- 3.5 Hilbert Spaces — 226
- 3.6 Bounded and Unbounded Linear Operators — 238
- 3.7 Compact Operators – Fredholm Operators — 252
- 3.8 Remarks — 268

4 Banach Spaces of Functions and Measures — 281

- 4.1 L^p -Spaces — 281
- 4.2 Lebesgue–Bochner Spaces — 300
- 4.3 Functions of Bounded Variations — 322
- 4.4 Absolutely Continuous Functions — 340
- 4.5 Sobolev Spaces — 351
- 4.6 Spaces of Measures — 362

4.7	Young Measures —	375
4.8	Remarks —	383
5	Convex Functions – Nonsmooth Analysis —	393
5.1	Convex Functions – Continuity Properties —	393
5.2	Differentiability of Convex Functions —	402
5.3	Conjugate Functions – Convex Subdifferential —	412
5.4	Proximal and Chebyshev Sets —	427
5.5	Smoothness of the Norm —	431
5.6	Multifunctions – Integral Functionals —	436
5.7	Lipschitz and Locally Lipschitz Functions —	460
5.8	Remarks —	473
6	Nonlinear Analysis —	483
6.1	Operators of Monotone Type —	483
6.2	Brouwer Degree —	511
6.3	Leray–Schauder Degree —	528
6.4	Fixed Point Theory —	536
6.4.1	Metric Fixed Point Theory —	537
6.4.2	Topological Fixed Point Theory —	547
6.5	Fixed Point Index —	553
6.6	Variational Principles —	561
6.7	Variational Convergence —	570
6.8	Remarks —	579
Bibliography —		587
Index —		599
List of Symbols —		609