

Pavel Drábek · Gabriela Holubová

Elements of Partial Differential Equations



Walter de Gruyter
Berlin · New York

Contents

Preface	v
1 Mathematical Models, Conservation and Constitutive Laws	1
1.1 Basic Notions	1
1.2 Evolution Conservation Law	3
1.3 Stationary Conservation Law	4
1.4 Conservation Law in One Dimension	4
1.5 Constitutive Laws	6
1.6 Exercises	8
2 Classification, Types of Equations, Boundary and Initial Conditions	9
2.1 Basic Types of Equations, Boundary and Initial Conditions	9
2.2 Classification of Linear Equations of the Second Order	14
2.3 Exercises	17
3 Linear Partial Differential Equations of the First Order	21
3.1 Convection and Transport Equation	21
3.2 Equations with Constant Coefficients	22
3.3 Equations with Non-Constant Coefficients	28
3.4 Exercises	32
4 Wave Equation in One Spatial Variable—Cauchy Problem in \mathbb{R}	37
4.1 String Vibrations and Wave Equation in One Dimension	37
4.2 Cauchy Problem on the Real Line	40
4.3 Wave Equation with Sources	49
4.4 Exercises	53
5 Diffusion Equation in One Spatial Variable—Cauchy Problem in \mathbb{R}	57
5.1 Diffusion and Heat Equations in One Dimension	57
5.2 Cauchy Problem on the Real Line	58
5.3 Diffusion Equation with Sources	65
5.4 Exercises	68
6 Laplace and Poisson Equations in Two Dimensions	71
6.1 Steady States and Laplace and Poisson Equations	71
6.2 Invariance of the Laplace Operator, Its Transformation into Polar Co-ordinates	73
6.3 Solution of Laplace and Poisson Equations in \mathbb{R}^2	74
6.4 Exercises	76

7	Solutions of Initial Boundary Value Problems for Evolution Equations	78
7.1	Initial Boundary Value Problems on Half-Line	78
7.2	Initial Boundary Value Problem on Finite Interval, Fourier Method	84
7.3	Fourier Method for Nonhomogeneous Problems	99
7.4	Transformation to Simpler Problems	103
7.5	Exercises	104
8	Solutions of Boundary Value Problems for Stationary Equations	113
8.1	Laplace Equation on Rectangle	113
8.2	Laplace Equation on Disc	115
8.3	Poisson Formula	117
8.4	Exercises	119
9	Methods of Integral Transforms	123
9.1	Laplace Transform	123
9.2	Fourier Transform	128
9.3	Exercises	134
10	General Principles	139
10.1	Principle of Causality (Wave Equation)	139
10.2	Energy Conservation Law (Wave Equation)	141
10.3	Ill-Posed Problem (Diffusion Equation for Negative t)	144
10.4	Maximum Principle (Heat Equation)	145
10.5	Energy Method (Diffusion Equation)	147
10.6	Maximum Principle (Laplace Equation)	148
10.7	Consequences of Poisson Formula (Laplace Equation)	150
10.8	Comparison of Wave, Diffusion and Laplace Equations	152
10.9	Exercises	152
11	Laplace and Poisson equations in Higher Dimensions	157
11.1	Invariance of the Laplace Operator	157
11.2	Green's First Identity	159
11.3	Properties of Harmonic Functions	161
11.4	Green's Second Identity and Representation Formula	164
11.5	Boundary Value Problems and Green's Function	166
11.6	Dirichlet Problem on Half-Space and on Ball	168
11.7	Exercises	174
12	Diffusion Equation in Higher Dimensions	178
12.1	Heat Equation in Three Dimensions	178
12.2	Cauchy Problem in \mathbb{R}^3	179
12.3	Diffusion on Bounded Domains, Fourier Method	182
12.4	Exercises	192

13 Wave Equation in Higher Dimensions	195
13.1 Membrane Vibrations and Wave Equation in Two Dimensions	195
13.2 Cauchy Problem in \mathbb{R}^3 —Kirchhoff's Formula	196
13.3 Cauchy problem in \mathbb{R}^2	199
13.4 Wave with sources in \mathbb{R}^3	202
13.5 Characteristics, Singularities, Energy and Principle of Causality	204
13.6 Wave on Bounded Domains, Fourier Method	208
13.7 Exercises	224
14 Appendix	228
14.1 Sturm-Liouville problem	228
14.2 Bessel Functions	229
Some Typical Problems Considered in This Book	235
Notation	237
Bibliography	239
Index	241