

Contents

Preface	v
Introduction	1
Glossary	3
1 Heegaard splittings	15
1.1 Introduction	15
1.2 Existence of Heegaard splittings	16
1.3 Stable equivalence of Heegaard splittings	16
1.4 The mapping class group	20
1.5 Manifolds of Heegaard genus ≤ 1	21
1.6 Seifert manifolds	25
2 Dehn surgery	27
2.1 Knots and links in 3-manifolds	27
2.2 Surgery on links in S^3	28
2.3 Surgery description of lens spaces and Seifert manifolds	30
2.4 Surgery and 4-manifolds	34
3 Kirby calculus	37
3.1 The linking number	37
3.2 Kirby moves	39
3.3 The linking matrix	48
3.4 Reversing orientation	49
4 Even surgeries	51
5 Review of 4-manifolds	57
5.1 Definition of the intersection form	57
5.2 The unimodular integral forms	60
5.3 Four-manifolds and intersection forms	62
6 Four-manifolds with boundary	65
6.1 The intersection form	65
6.2 Homology spheres via surgery on knots	69
6.3 Seifert homology spheres	70
6.4 The Rohlin invariant	72

7	Invariants of knots and links	73
7.1	Seifert surfaces	73
7.2	Seifert matrices	75
7.3	The Alexander polynomial	77
7.4	Other invariants from Seifert surfaces	81
7.5	Knots in homology spheres	83
7.6	Boundary links and the Alexander polynomial	84
8	Fibered knots	87
8.1	The definition of a fibered knot	87
8.2	The monodromy	88
8.3	More about torus knots	91
8.4	Joins	92
8.5	The monodromy of torus knots	94
9	The Arf-invariant	97
9.1	The Arf-invariant of a quadratic form	97
9.2	The Arf-invariant of a knot	99
10	Rohlin's theorem	103
10.1	Characteristic surfaces	103
10.2	The definition of $\tilde{q}$	104
10.3	Representing homology classes by surfaces	108
11	The Rohlin invariant	111
11.1	Definition of the Rohlin invariant	111
11.2	The Rohlin invariant of Seifert spheres	111
11.3	A surgery formula for the Rohlin invariant	114
11.4	The homology cobordism group	117
12	The Casson invariant	121
13	The group $SU(2)$	129
14	Representation spaces	135
14.1	The topology of representation spaces	135
14.2	Irreducible representations	136
14.3	Representations of free groups	136
14.4	Representations of surface groups	137
14.5	Representations of Seifert homology spheres	139
15	The local properties of representation spaces	145

16 Casson's invariant for Heegaard splittings	149
16.1 The intersection product	149
16.2 The orientations	151
16.3 Independence of Heegaard splitting	154
17 Casson's invariant for knots	157
17.1 Preferred Heegaard splittings	157
17.2 The Casson invariant for knots	158
17.3 The difference cycle	161
17.4 The Casson invariant for unlinks	163
17.5 The Casson invariant of a trefoil	164
18 An application of the Casson invariant	167
18.1 Triangulating 4-manifolds	167
18.2 Higher-dimensional manifolds	168
19 The Casson invariant of Seifert manifolds	169
19.1 The space $\mathcal{R}(p, q, r)$	169
19.2 Calculation of the Casson invariant	172
Conclusion	177
Exercises	183
Bibliography	187
Index	197