

Contents

Preface *IX*

1	The MHD Equations	1
1.1	Derivation of the MHD Equations	1
1.1.1	Multispecies MHD Equations	1
1.1.2	One-Fluid Model of Magnetohydrodynamics	4
1.1.3	Validity of the One-Fluid Model of Magnetohydrodynamics	6
1.2	Consequences of the MHD Equations	8
1.2.1	Magnetic Flux Conservation	8
1.2.2	MHD Equilibrium	10
1.2.3	Magnetohydrodynamic Waves	11
1.2.3.1	Compressional Alfvén Waves	12
1.2.3.2	Shear Alfvén Waves	13
2	MHD Equilibria in Fusion Plasmas	15
2.1	Linear Configurations	15
2.1.1	The z-Pinch	15
2.1.2	The Screw Pinch	18
2.2	Toroidal Configurations	22
2.2.1	The Tokamak	23
2.2.1.1	The Grad–Shafranov Equation	23
2.2.1.2	Circular Cross Section	27
2.2.1.3	Arbitrary Cross Section	32
2.2.1.4	The Straight Field Line Angle	34
2.2.2	The Stellarator	37
3	Linear Ideal MHD Stability Analysis	43
3.1	Linear MHD Stability as an Initial Value Problem	44
3.2	The Energy Principle of Ideal MHD	47
3.3	Forms of δW	48
3.4	The Ideal MHD Energy Principle for the Tokamak	51

4	Current Driven Ideal MHD Modes in a Tokamak	55
4.1	Expression for δW in Tokamak Ordering	55
4.2	External Kinks in a Tokamak with $\beta = 0$	56
4.2.1	Modes with $m = 1$	56
4.2.2	Modes with $m \geq 2$	58
4.3	Internal Kink Modes	61
4.4	$n = 0$ Modes: The Vertical Displacement Event (VDE)	63
5	Pressure Driven Modes in a Tokamak	69
5.1	Localized Interchange Modes in the Screw Pinch	69
5.2	Localized Pressure Driven Modes in the Tokamak	72
5.2.1	Interchange Modes in a Tokamak	73
5.2.2	Ballooning Modes	76
6	Combined Pressure and Current Driven Modes: Edge Localized Modes	83
6.1	ELM Phenomenology	84
6.2	Linear Stability of the Pedestal	86
6.3	Non-linear Evolution	90
6.3.1	Non-linear Cycles	90
6.3.2	Magnitude of the ELM Crash	92
6.3.3	Timescale of the ELM Crash	94
6.4	ELM Control	94
6.4.1	Small ELM Regimes	95
6.4.2	Active ELM Control	97
7	Combined Pressure and Current Driven Modes: The Ideal β-Limit	103
7.1	Tokamak Operational Scenarios	103
7.2	External Kink Modes in a Tokamak with Finite β	105
7.3	The Effect of a Conducting Wall on External Kink Modes	107
7.3.1	Ideally Conducting Wall	107
7.3.2	Resistive Wall	110
7.4	The Resistive Wall Mode (RWM)	112
7.5	The Troyon Limit	118
8	Resistive MHD Stability	123
8.1	Stability of Current Sheets	124
8.2	Reconnection in the Presence of a Guide Field	127
8.3	Magnetic Islands in Tokamaks	134
8.4	The Rutherford Equation	137
9	Current Driven ('classical') Tearing Modes in Tokamaks	141
9.1	Effect of Tearing Modes on Kinetic Profiles	141
9.2	Nonlinear Saturation	144
9.3	Tearing Mode Rotation and Locking	146

9.3.1	Rotation of Tearing Modes in Tokamaks	146
9.3.2	Locking of Pre-existing Magnetic Islands	148
9.3.3	<i>Ab-initio</i> Locked Modes	152
10	Disruptions	159
10.1	Phenomenology of Disruptions	159
10.1.1	The Density Limit	161
10.2	Consequences of Disruptions	165
10.2.1	Thermal Loads	165
10.2.2	Mechanical Loads	166
10.2.3	Runaway Generation	168
10.3	Disruption Avoidance and Mitigation	171
11	M=1 Modes beyond Ideal MHD: Sawteeth and Fishbones	175
11.1	The Sawtooth Instability	175
11.1.1	Phenomenology	175
11.1.2	Sawtooth Period and Onset Criterion	176
11.1.3	Models for the Sawtooth Crash	181
11.2	The Fishbone Instability	184
12	Tearing Modes in Finite β-Tokamaks	189
12.1	The Modified Rutherford Equation	189
12.2	The Neoclassical Tearing Mode (NTM)	190
12.3	Onset Criteria for NTMs	194
12.4	Frequently Interrupted Regime (FIR) NTMs	197
13	Control of Resistive MHD Instabilities by External Current Drive	201
13.1	Basic Properties of Localized Electron Cyclotron Current Drive (ECCD)	202
13.2	Criteria for Control of Resistive Instabilities	203
13.2.1	Control by Changing the Equilibrium Current Density	203
13.2.2	Control by Generating Helical Currents	206
13.3	Sawtooth Control	208
13.4	Tearing Mode Control	211

References 215

Index 221