C. Truesdell · W. Noll

The Non-Linear Field Theories of Mechanics

Edited by Stuart S. Antman

Third Edition



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