

# Table of Contents

## Part I: One-dimensional, One-channel Systems

<b>Chapter 1 The principal Equations of Scattering Theory</b> . . . . .	1
1.1 General Remarks . . . . .	1
1.2 Elements of the Direct and Inverse Problems . . . . .	2
1.2.1 The Simplest Difference Schrödinger Equation . . . . .	2
1.2.2 Potential Wells of Infinite Depth . . . . .	3
1.2.3 The Direct Problem . . . . .	4
1.2.4 The Inverse Problem . . . . .	6
1.2.5 Scattering by a Potential of Finite Range . . . . .	7
1.2.6 The Finite-Difference Analogue of the $R$ -matrix Scattering Theory . . . . .	9
1.2.7 Conditions of Orthonormality and Completeness of the Eigenfunctions of the Finite-Difference Schrödinger Operator on $[0, \alpha]$ . . . . .	10
1.2.8 Relations Between the Scattering Parameters $\{E_\lambda \text{ and } \gamma_\lambda\}$ . . . . .	11
1.2.9 Reconstruction of the Potential on the Semi-axis $0 \leq x < \infty$ . . . . .	11
1.3 The Gelfand-Levitan-Marchenko Equations . . . . .	12
1.3.1 The Regular Solutions $\varphi$ and $\hat{\varphi}$ . . . . .	12
1.3.2 The Algebraic Analogue of the Gelfand-Levitan Equations . . . . .	14
1.3.3 The Relationship Between $K(n, m)$ and the Potential $V(n)$ . . . . .	16
1.3.4 The Gelfand-Levitan Formalism ( $\Delta \rightarrow dx$ ) . . . . .	16
1.3.5 The $R$ -matrix Inverse Problem . . . . .	17
1.3.6 The Inverse Problem on the Semi-axis Within the Gelfand-Levitan Approach . . . . .	18
1.3.7 The Algebraic Analogue of the Marchenko Method . . . . .	19
1.3.8 The Marchenko Equations for $\Delta \rightarrow dx$ . . . . .	22
1.3.9 Relations Between $V(n)$ and $K(m, n)$ in the Marchenko Approach . . . . .	23
1.4 Miscellaneous Direct and Inverse Problems . . . . .	24
1.4.1 Stationary Solutions and Wave Propagation . . . . .	25
1.4.2 Penetrability of Potential Barriers . . . . .	29
1.4.3 The Inverse Problem on the Whole Axis . . . . .	34
1.4.4 Reconstruction of Potentials from Resonance Parameters . . . . .	41
1.5 Notes on the Literature . . . . .	50
1.6 Exercises . . . . .	53

<b>Chapter 2 Exactly Solvable Models: Bargmann Potentials <math>V^B</math></b>	55
2.1 General Comments	55
2.2 Simplest Examples of $V^B$ ( $\sigma \leq \tau \infty$ ) for $l = 0$	56
2.2 Potentials with a Single Bound State	57
2.2.2 Potentials Without Bond States: $S(k)$ with a Single Pole in the Upper $k$ Walf-plane	60
2.2.3 Potentials with $S(k)$ with Two Poles in the Upper $k$ Half-Plane	61
2.3 More General Models	63
2.3 Multi-term Degenerate Kernels of the Inverseproblem Equations	65
2.3.2 Models of One-Dimensional Motion on the Whole Axis	69
2.3.3 The Finite-Difference-Approach	69
2.3.4 The Rational Reflection Coefficient (no Bound States)	70
2.3.5 The Finite-difference Approach	71
2.4 Potentials of the Finite-Range and Infinitely Deep Wells.	
R-matrix Models	72
2.5 Potentials Allowing Exact Solutions for Variable Angular Momenta	83
2.5.1 Potentials from Spectral Data at Fixed Energy and Variable $l$	83
2.5.2 Newton-Sabatier Potentials	84
2.5.3 The Generalized Crum-Krein Transformations	85
2.5.4 Lipperheide-Fidelday Potentials	88
2.6 Notes on the Literature	89
2.7 Exercises	91
<b>Chapter 3 Approximate Solutions</b>	92
3.1 Convergence of the Approximations, Stability and Regularization of Solutions	93
3.2 Solutions Using Bargmann Potentials	
3.4 Approximation of Datentials by Steps, at Discrete Points, and by Splines.	
The Role of the Upper Part of the Spectrum	109
3.5 Method of Multiple Solutions of the Direct Problem	121
3.6 Notes on the Literature	122
<b>Chapter 4 The Levinson Theorem</b>	125
4.1 General Remarks	125
4.2 Simple Examples	126
4.3 The Coulomb Potential and Other Singular Interactions	130
4.4 Other Types of Interactions	134
4.4.1 Potentials Depending on Velocity	134
4.4.2 Potentials Depending on Energy	134
4.4.3 The Finite-Difference Schrödinger Equation	135
4.4.4 Motion Along the Axis	135
4.4.5 Nonlocal Potentials	136
4.5 Notes on the Literature	137

**Part II. Multi-channel, Multi-dimensional, Multi-particle Problems**

<b>Chapter 5 Multi-channel Equations</b> . . . . .	140
5.1 General Remarks . . . . .	140
5.2 The Formalism of Multi-channel Coupling . . . . .	140
5.3 Finite-Difference Equations of Motion . . . . .	145
5.4 Exactly Solvable Models . . . . .	149
5.5 Notes on the Literature . . . . .	151
5.6 Exercises . . . . .	152
<b>Chapter 6 Multi-dimensional Problems</b>	
6.1 General Remarks . . . . .	153
6.2 The Finite-Difference Formalism . . . . .	155
6.3 Reduction of Multi-dimensional problems to Multichannel problems . . . . .	159
6.4 The Multi-dimensional Inverse Problem . . . . .	167
6.5 Separation of Variables in Noncentral Field . . . . .	168
6.6 Exactly Solvable Models . . . . .	170
6.7 Notes on the Literature . . . . .	176
<b>Chapter 7 Multi-particle Systems</b> . . . . .	177
7.1 General Remarks . . . . .	177
7.2 Asymptotic Hamiltonians and Boundary Conditions . . . . .	178
7.3 Tunnelling Through Potential Barriers by Several Particles . . . . .	182
7.4 Excitation of Collective Degrees of Freedom of Multi-particle Systems . . . . .	188
7.4.1 Transformation of Amplitudes in Transition to the Reference Frame of the Target . . . . .	
7.5 The Method of Hyperspherical Functions ( $K$ -harmonics) . . . . .	191
7.6 The Levinson Theorem . . . . .	196
7.7 Three-Particle Potentials . . . . .	197
7.8 Notes on the Literature . . . . .	202
<b>References</b> . . . . .	210
<b>Subject Index</b> . . . . .	221