Waves Called Solitons

Concepts and Experiments

3rd Revised and Enlarged Edition With 170 Figures



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