

Contents

1	Krichever–Novikov algebras: basic definitions and structure theory	1
1.1	Current, vector field, and other Krichever–Novikov algebras	1
1.2	Meromorphic λ -forms and Krichever–Novikov duality	2
1.3	Krichever–Novikov bases	4
1.4	Almost-graded structure, triangle decompositions	6
1.5	Central extensions and 2-cohomology; Virasoro-type algebras	9
1.6	Affine Krichever–Novikov, in particular Kac–Moody, algebras	13
1.7	Central extensions of the Lie algebra \mathcal{D}_g^1	15
1.8	Local cocycles for $\mathfrak{sl}(n)$ and $\mathfrak{gl}(n)$	16
2	Fermion representations and Sugawara construction	19
2.1	Admissible representations and holomorphic bundles	19
2.2	Holomorphic bundles in the Tyurin parametrization	21
2.3	Krichever–Novikov bases for holomorphic vector bundles	23
2.4	Fermion representations of affine algebras	26
2.5	Verma modules for affine algebras	29
2.6	Fermion representations of Virasoro-type algebras	31
2.7	Sugawara representation	34
2.8	Proof of the main theorems for the Sugawara construction	39
2.8.1	Main theorems in the form of relations with structure constants	40
2.8.2	End of the proof of the main theorems	43
3	Projective flat connections on the moduli space of punctured Riemann surfaces and the Knizhnik–Zamolodchikov equation	55
3.1	Virasoro-type algebras and moduli spaces of Riemann surfaces	56
3.2	Sheaf of conformal blocks and other sheaves on the moduli space	
	$\mathcal{M}_{g,N+1}^{(1,0)}$	62

3.3	Differentiation of the Krichever–Novikov objects in modular variables	63
3.4	Projective flat connection and generalized Knizhnik–Zamolodchikov equation	67
3.5	Explicit form of the Knizhnik–Zamolodchikov equations for genus 0 and genus 1	72
3.5.1	Explicit form of the equations for $g = 0$	72
3.5.2	Explicit form of the equations for $g = 1$	76
3.6	Appendix: the Krichever–Novikov base in the elliptic case	81
4	Lax operator algebras	84
4.1	Lax operators and their Lie bracket	85
4.1.1	Lax operator algebras for $\mathfrak{gl}(n)$ and $\mathfrak{sl}(n)$	85
4.1.2	Lax operator algebras for $\mathfrak{so}(n)$	86
4.1.3	Lax operator algebras for $\mathfrak{sp}(2n)$	88
4.2	Almost-graded structure	90
4.3	Central extensions of Lax operator algebras: the construction	92
4.4	Uniqueness theorem	98
5	Lax equations on Riemann surfaces, and their hierarchies	101
5.1	M -operators	103
5.2	L -operators and Lax operator algebras from M -operators	106
5.3	g -valued Lax equations	107
5.4	Hierarchies of commuting flows	111
5.5	Symplectic structure	113
5.6	Hamiltonian theory	117
5.7	Examples: Calogero–Moser systems	124
6	Lax integrable systems and conformal field theory	129
6.1	Conformal field theory related to a Lax integrable system	129
6.2	From Lax operator algebra to commutative Krichever–Novikov algebra	131
6.3	The representation of \mathcal{A}_L	132
6.4	Sugawara representation	134
6.5	Conformal blocks and the Knizhnik–Zamolodchikov connection	135

6.6	The representation of the algebra of Hamiltonian vector fields and commuting Hamiltonians	135
6.7	Unitarity	136
6.8	Relation to geometric quantization and quantum integrable systems ..	138
6.9	Remark on the Seiberg–Witten theory	138
	Bibliography	141
	Notation	147
	Index	149