

Contents

1	Prolog	1
1.1	Introduction	1
1.2	General Hints to the Literature	8
1.3	Notation and Conventions	10

Part I Foundations of Stochastic Geometry

2	Random Closed Sets	17
2.1	Random Closed Sets in Locally Compact Spaces	17
2.2	Characterization of Capacity Functionals	22
2.3	Some Consequences of Choquet's Theorem	31
2.4	Random Closed Sets in Euclidean Space	37
3	Point Processes	47
3.1	Random Measures and Point Processes	48
3.2	Poisson Processes	58
3.3	Palm Distributions	70
3.4	Palm Distributions – General Approach	79
3.5	Marked Point Processes	82
3.6	Point Processes of Closed Sets	95
4	Geometric Models	99
4.1	Particle Processes	100
4.2	Germ-grain Processes	109
4.3	Germ-grain Models, Boolean Models	117
4.4	Processes of Flats	124
4.5	Surface Processes	140
4.6	Associated Convex Bodies	145

Part II Integral Geometry

5	Averaging with Invariant Measures	167
5.1	The Kinematic Formula for Additive Functionals	168
5.2	Translative Integral Formulas	180
5.3	The Principal Kinematic Formula for Curvature Measures	190
5.4	Intersection Formulas for Submanifolds	203
6	Extended Concepts of Integral Geometry	211
6.1	Rotation Means of Minkowski Sums	211
6.2	Projection Formulas	220
6.3	Cylinders and Thick Sections	223
6.4	Translative Integral Geometry, Continued	228
6.5	Spherical Integral Geometry	248
7	Integral Geometric Transformations	265
7.1	Flag Spaces	266
7.2	Blaschke–Petkantschin Formulas	270
7.3	Transformation Formulas Involving Spheres	287

Part III Selected Topics from Stochastic Geometry

8	Some Geometric Probability Problems	293
8.1	Historical Examples	293
8.2	Convex Hulls of Random Points	298
8.3	Random Projections of Polytopes	328
8.4	Randomly Moving Bodies and Flats	335
8.5	Touching Probabilities	349
8.6	Extremal Problems for Probabilities and Expectations	359
9	Mean Values for Random Sets	377
9.1	Formulas for Boolean Models	379
9.2	Densities of Additive Functionals	393
9.3	Ergodic Densities	404
9.4	Intersection Formulas and Unbiased Estimators	413
9.5	Further Estimation Problems	429
10	Random Mosaics	445
10.1	Mosaics as Particle Processes	446
10.2	Voronoi and Delaunay Mosaics	470
10.3	Hyperplane Mosaics	484
10.4	Zero Cells and Typical Cells	493
10.5	Mixing Properties	515

11 Non-stationary Models	521
11.1 Particle Processes and Boolean Models	522
11.2 Contact Distributions	534
11.3 Processes of Flats	543
11.4 Tessellations	550

Part IV Appendix

12 Facts from General Topology	559
12.1 General Topology and Borel Measures	559
12.2 The Space of Closed Sets	563
12.3 Euclidean Spaces and Hausdorff Metric	570
13 Invariant Measures	575
13.1 Group Operations and Invariant Measures	575
13.2 Homogeneous Spaces of Euclidean Geometry	581
13.3 A General Uniqueness Theorem	593
14 Facts from Convex Geometry	597
14.1 The Subspace Determinant	597
14.2 Intrinsic Volumes and Curvature Measures	599
14.3 Mixed Volumes and Inequalities	610
14.4 Additive Functionals	617
14.5 Hausdorff Measures and Rectifiable Sets	633
References	637
Author Index	675
Subject Index	681
Notation Index	689