

Table of Contents

Preface	xi
Introduction	xiii
Chapter 1 Orthogonal Polynomials and Krein's Theorem	1
1.1 Szegő's Theorem	1
1.2 Krein's Theorem	6
1.3 Restatement of Krein's Theorem	13
1.4 One-Step Version of Krein's Theorem	15
1.5 Proof of Krein's Theorem	23
1.6 Continuous Analogue of Krein's Theorem	25
1.7 Comments to Chapter 1	27
Chapter 2 Reformulations of Krein's Theorem	29
2.1 Krein's Theorem in Terms of Infinite Toeplitz Matrices	29
2.2 Krein's Theorem in Terms of Modules	33
2.3 Comments to Chapter 2	35
Chapter 3 Inner Products on Modules and Orthogonalization with Invertible Squares	37
3.1 Inner Products on Modules Over C^* -Algebras	37
3.2 Orthogonalization with Invertible Squares	40
3.3 Orthogonalization in the Module $L_2^{r \times r}(\mathbb{T})$	46
3.4 Orthogonalization in the Module $\mathcal{M}_R$	48
3.5 Comments to Chapter 3	51
Chapter 4 Orthogonal Matrix Polynomials	53
4.1 Introduction	53
4.2 An Inertia Theorem	55
4.3 Krein's Theorem for Matrix Polynomials	58
4.4 The Positive Definite Case	63
4.5 Reformulation of Krein's Theorem for Matrix Polynomials	65
4.6 Comments to Chapter 4	70
Chapter 5 Special Class of Block Toeplitz Matrices	71
5.1 Preliminaries	72
5.2 An Identity	73

5.3	Matrix Version of Rouché's Theorem	78
5.4	Special Form of Krein's Theorem	80
5.5	Example	85
5.6	Comments to Chapter 5	86
Chapter 6 Orthogonal Operator-Valued Polynomials:		
	First Generalization	87
6.1	Generalization of Krein's Theorem	87
6.2	Reformulation of the Main Theorem	93
6.3	Comments to Chapter 6	98
Chapter 7 Convolution Equations on a Finite Interval		99
7.1	Convolution Operators on a Finite Interval	99
7.2	Invertibility Conditions for Convolution Operators on a Finite Interval	101
7.3	The Inversion Formula	106
7.4	Comments to Chapter 7	114
Chapter 8 Continuous Analogues of Orthogonal Matrix Polynomials		115
8.1	The Main Theorem	115
8.2	An Identity	117
8.3	Discretization	122
8.4	Wiener-Hopf Operators	129
8.5	Proof of the Main Theorem	132
8.6	Reformulation of Krein's Theorem for Continuous Analogues of Orthogonal Matrix Polynomials	139
8.7	Examples of Continuous Analogues of Orthogonal Matrix Polynomials	140
8.8	Comments to Chapter 8	142
Chapter 9 Orthogonal Operator-Valued Polynomials		143
9.1	Generalized Inertia Theorems	143
9.2	Orthogonal Operator-Valued Polynomials	148
9.3	Comments to Chapter 9	158
Appendix to Chapter 9 Time-Variant Analogues of Orthogonal Polynomials		159
A.1	Inertia Theorem	159
A.2	Time-Variant Analogue of Krein's Theorem	160
A.3	Comments to the Appendix	161

Chapter 10	Reverse, Left and Right Orthogonalization	163
10.1	Reverse Orthogonalization with Respect to a Scalar Product	163
10.2	Left and Right Orthogonalization of Two-Sided Sequences	173
10.3	Reverse Orthogonalization with Respect to an Inner Product	176
10.4	Comments to Chapter 10	182
Chapter 11	Discrete Infinite Analogue of Krein's Theorem	183
11.1	Preliminaries	184
11.2	Left and Right Orthogonalization	189
11.3	Identities	194
11.4	Analogue of Krein's Theorem	198
11.5	Comments to Chapter 11	203
Chapter 12	Continuous Infinite Analogue of Krein's Theorem	205
12.1	Statement of Main Theorem	205
12.2	Auxilliary Theorems	208
12.3	Proof of Main Theorem	218
12.4	Comments to Chapter 12	226
References		227
Index of Symbols		233
Index		235