

# Contents

|                                                                                                          |    |
|----------------------------------------------------------------------------------------------------------|----|
| <b>Introduction</b>                                                                                      | 1  |
| <b>1 Rate Equation Approach for Growing Networks</b>                                                     |    |
| <i>P.L. Krapivsky, S. Redner</i>                                                                         | 3  |
| 1.1 Introduction                                                                                         | 3  |
| 1.2 Structure of the Growing Network                                                                     | 4  |
| 1.2.1 The Degree Distribution                                                                            | 4  |
| 1.2.2 Order Distribution                                                                                 | 7  |
| 1.2.3 Degree Correlations                                                                                | 8  |
| 1.2.4 Global Properties                                                                                  | 9  |
| 1.3 Finiteness, Fluctuations, and Extremes                                                               | 11 |
| 1.3.1 Role of Finiteness                                                                                 | 11 |
| 1.3.2 Extremes and Lead Changes                                                                          | 13 |
| 1.4 Protein Networks                                                                                     | 16 |
| 1.4.1 Infinite-Order Percolation Transition                                                              | 16 |
| 1.4.2 Non-universal Degree Distribution                                                                  | 19 |
| 1.5 Outlook                                                                                              | 21 |
| References                                                                                               | 21 |
| <b>2 Directed and Non-directed Scale-Free Networks</b>                                                   |    |
| <i>Reuven Cohen, Alejandro Fabian Rozenfeld, Nehemia Schwartz,<br/>Daniel ben-Avraham, Shlomo Havlin</i> | 23 |
| 2.1 Random Scale-Free Networks                                                                           | 23 |
| 2.1.1 Percolation Threshold                                                                              | 23 |
| 2.1.2 Generating Functions                                                                               | 27 |
| 2.1.3 Critical Exponents                                                                                 | 28 |
| 2.2 Directed Graphs                                                                                      | 28 |
| 2.2.1 Structure                                                                                          | 29 |
| 2.2.2 Percolation Threshold                                                                              | 30 |
| 2.2.3 Critical Exponents                                                                                 | 33 |
| 2.2.4 Summary                                                                                            | 36 |
| 2.3 Spatially Embedded Scale-Free Graphs                                                                 | 37 |
| 2.3.1 Model Definition                                                                                   | 38 |
| 2.3.2 Summary                                                                                            | 43 |
| References                                                                                               | 44 |

**3 Hierarchical Organization of Modularity in Complex Networks**

|                                                                                                         |    |
|---------------------------------------------------------------------------------------------------------|----|
| <i>Albert-László Barabási, Erzsébet Ravasz, Zoltán Oltvai</i> .....                                     | 46 |
| 3.1 Introduction .....                                                                                  | 46 |
| 3.2 Hierarchical Network Model .....                                                                    | 49 |
| 3.3 Hierarchical Organization in Non-biological Networks .....                                          | 52 |
| 3.4 Hierarchy in Metabolic Networks and the Functional Organization<br>of <i>Escherichia Coli</i> ..... | 55 |
| 3.5 Stochastic Model and Universality .....                                                             | 60 |
| 3.6 Discussion and Outlook .....                                                                        | 62 |
| References .....                                                                                        | 64 |

**4 Mixing Patterns and Community Structure in Networks**

|                                                                 |    |
|-----------------------------------------------------------------|----|
| <i>M.E.J. Newman, M. Girvan</i> .....                           | 66 |
| 4.1 Introduction .....                                          | 66 |
| 4.2 Community Structure .....                                   | 67 |
| 4.2.1 Edge Betweenness and Community Detection .....            | 69 |
| 4.2.2 Examples .....                                            | 71 |
| 4.3 Origins of Community Structure and Assortative Mixing ..... | 73 |
| 4.4 Other Types of Assortative Mixing .....                     | 78 |
| 4.4.1 Mixing by Vertex Degree .....                             | 79 |
| 4.5 Conclusions .....                                           | 84 |
| References .....                                                | 85 |

**5 Effect of Accelerated Growth on Networks Dynamics**

|                                                                       |     |
|-----------------------------------------------------------------------|-----|
| <i>J.F.F. Mendes</i> .....                                            | 88  |
| 5.1 The Meaning of Acceleration in Networks .....                     | 88  |
| 5.2 Degree Distributions .....                                        | 91  |
| 5.2.1 What Types of Degree Distribution Can We Have? .....            | 91  |
| 5.2.2 The Most Interesting Case: Power-Law Degree Distribution ...    | 93  |
| 5.3 General Relations for the Accelerated Growth .....                | 98  |
| 5.4 Scaling Relations for Accelerated Growth .....                    | 100 |
| 5.5 What Are the Degree Distributions Produced by Acceleration? ..... | 101 |
| 5.5.1 Model for $\gamma < 2$ .....                                    | 101 |
| 5.5.2 Model for $\gamma > 2$ .....                                    | 102 |
| 5.6 One Practical Example: The Word Web .....                         | 105 |
| 5.7 Conclusions .....                                                 | 109 |
| References .....                                                      | 110 |

**6 Optimization in Complex Networks**

|                                                    |     |
|----------------------------------------------------|-----|
| <i>Ramon Ferrer i Cancho, Ricard V. Solé</i> ..... | 114 |
| 6.1 Introduction .....                             | 114 |
| 6.2 Network Optimization .....                     | 115 |
| 6.3 The Optimization Algorithm .....               | 116 |
| 6.4 Optimal Degree Distributions .....             | 119 |
| 6.5 Discussion .....                               | 122 |
| References .....                                   | 125 |

## 7 Epidemic Spreading in Complex Networks with Degree Correlations

|                                                                                 |     |
|---------------------------------------------------------------------------------|-----|
| <i>Marián Boguñá, Romualdo Pastor-Satorras, Alessandro Vespignani</i> . . . . . | 127 |
| 7.1 Introduction . . . . .                                                      | 127 |
| 7.2 Correlated Complex Networks . . . . .                                       | 128 |
| 7.2.1 Assortative and Disassortative Mixing . . . . .                           | 129 |
| 7.2.2 Degree Detailed Balance Condition . . . . .                               | 130 |
| 7.3 The SIS Model . . . . .                                                     | 133 |
| 7.3.1 Uncorrelated Homogeneous Networks . . . . .                               | 133 |
| 7.3.2 Uncorrelated Complex Networks . . . . .                                   | 134 |
| 7.3.3 Correlated Complex Networks . . . . .                                     | 135 |
| 7.3.4 Correlated Scale-Free Networks . . . . .                                  | 136 |
| 7.4 The SIR Model . . . . .                                                     | 138 |
| 7.4.1 Uncorrelated Homogeneous Networks . . . . .                               | 139 |
| 7.4.2 Uncorrelated Complex Networks . . . . .                                   | 140 |
| 7.4.3 Correlated Complex Networks . . . . .                                     | 142 |
| 7.4.4 Correlated Scale-Free Networks . . . . .                                  | 144 |
| 7.5 Conclusions . . . . .                                                       | 144 |
| References . . . . .                                                            | 145 |

## 8 Food Web Structure and the Evolution of Complex Networks

|                                                                           |     |
|---------------------------------------------------------------------------|-----|
| <i>Guido Caldarelli, Diego Garlaschelli, Luciano Pietronero</i> . . . . . | 148 |
| 8.1 Introduction . . . . .                                                | 148 |
| 8.2 Network Analysis . . . . .                                            | 150 |
| 8.2.1 Ecological Properties . . . . .                                     | 150 |
| 8.2.2 Small-World Properties . . . . .                                    | 151 |
| 8.2.3 Degree Distribution . . . . .                                       | 151 |
| 8.3 Spanning Tree Analysis . . . . .                                      | 152 |
| 8.3.1 Foodwebs as Transportation Networks . . . . .                       | 153 |
| 8.3.2 Allometric Scaling Relations . . . . .                              | 153 |
| 8.3.3 Efficiency of Empirical Food Webs . . . . .                         | 154 |
| 8.3.4 Stability under Species Removal . . . . .                           | 156 |
| 8.4 The Webworld Model . . . . .                                          | 157 |
| 8.4.1 Coupling between Topology and Dynamics . . . . .                    | 158 |
| 8.4.2 Initial State of the Model . . . . .                                | 158 |
| 8.4.3 Evolution of the Model . . . . .                                    | 160 |
| 8.4.4 Further Comments on the Model . . . . .                             | 164 |
| 8.5 Discussion and Conclusion . . . . .                                   | 165 |
| References . . . . .                                                      | 166 |

## 9 Social Networks: From Sexual Networks to Threatened Networks

|                                                             |     |
|-------------------------------------------------------------|-----|
| <i>H. Eugene Stanley, Shlomo Havlin</i> . . . . .           | 167 |
| 9.1 Introduction . . . . .                                  | 167 |
| 9.2 Recent Advances on Scale-Free Social Networks . . . . . | 168 |

|       |                                                         |     |
|-------|---------------------------------------------------------|-----|
| 9.3   | Recent Advances on Traffic Flow in Networks .....       | 169 |
| 9.4   | Characteristic Properties of Real Networks .....        | 169 |
| 9.4.1 | Classification of Real Networks .....                   | 169 |
| 9.4.2 | Percolation .....                                       | 170 |
| 9.4.3 | Structural and Transport Properties of Networks .....   | 170 |
| 9.4.4 | Optimizing the Stability of Threatened Networks .....   | 170 |
| 9.4.5 | Immunization of Networks .....                          | 171 |
| 9.5   | Possible Contributions of Social Network Research ..... | 171 |
| 9.6   | Discussion .....                                        | 172 |
|       | References .....                                        | 172 |

## 10 Search and Congestion in Complex Networks

|                                                                                                       |                                                    |     |
|-------------------------------------------------------------------------------------------------------|----------------------------------------------------|-----|
| <i>Alex Arenas, Antonio Cabrales, Albert Díaz-Guilera, Roger Guimerà, Fernando Vega-Redondo</i> ..... |                                                    | 175 |
| 10.1                                                                                                  | Introduction .....                                 | 175 |
| 10.2                                                                                                  | Search in Complex Networks .....                   | 176 |
| 10.3                                                                                                  | Load and Congestion in Complex Networks .....      | 178 |
| 10.4                                                                                                  | A Model of Communication .....                     | 179 |
| 10.4.1                                                                                                | Description of the Model .....                     | 179 |
| 10.4.2                                                                                                | Congestion and Network Capacity .....              | 181 |
| 10.5                                                                                                  | Analytical Results for Hierarchical Lattices ..... | 182 |
| 10.6                                                                                                  | Optimization in Model Networks .....               | 184 |
| 10.6.1                                                                                                | Network Topology .....                             | 185 |
| 10.6.2                                                                                                | Communication Model and Search Algorithm .....     | 186 |
| 10.6.3                                                                                                | Results .....                                      | 187 |
| 10.7                                                                                                  | Optimization in a General Framework .....          | 190 |
| 10.8                                                                                                  | Summary .....                                      | 193 |
|                                                                                                       | References .....                                   | 193 |

## 11 Membrane Clusters of Ion Channels: Size Effects for Stochastic Resonance

|                                                         |                                                      |     |
|---------------------------------------------------------|------------------------------------------------------|-----|
| <i>Gerhard Schmid, Igor Goychuk, Peter Hänggi</i> ..... |                                                      | 195 |
| 11.1                                                    | Introduction .....                                   | 195 |
| 11.2                                                    | The Hodgkin-Huxley Model .....                       | 196 |
| 11.3                                                    | Stochastic Version of the Hodgkin-Huxley Model ..... | 199 |
| 11.3.1                                                  | Quantifying Channel Noise .....                      | 200 |
| 11.3.2                                                  | Stochastic Resonance .....                           | 202 |
| 11.4                                                    | Conclusions .....                                    | 204 |
|                                                         | References .....                                     | 205 |