

Contents

Preface — IX

1	Number systems — 1
1.1	Sets — 1
1.2	Domains — 2
1.3	Assumptions in Mathematica® — 5
1.4	Quantifiers — 7
1.5	Complex numbers — 10
1.6	Real numbers — 11
1.7	Infinities — 13
1.8	Integers and the Principle of Mathematical Induction — 15
1.8.1	Example — 16
1.8.2	Example — 16
1.9	Algebraic equations and algebraic numbers — 18
1.10	Non-algebraic equations — 21
1.11	Sequences of real numbers and their limits — 22
1.11.1	Example — 25
1.11.2	Example: the number e — 25
1.12	Supremum and infimum — 27
1.12.1	Example — 29
2	Recursive sequences, discrete dynamical systems and their limits — 33
2.1	Example — 34
2.2	Example: the Fibonacci sequence — 41
3	Series — 47
3.1	Sequences and series — 47
3.2	The functions Sum and NSum — 49
3.3	Absolute convergence — 51
3.4	Convergence of series with terms of constant signs — 52
3.4.1	Example — 53
3.5	Convergence of series with terms of non-constant signs — 55
3.5.1	Grouping of terms — 55
3.5.2	Example — 56
3.5.3	Abel's summation formula — 57
3.5.4	Dirichlet's and Abel's tests — 58
3.5.5	Example — 60
3.6	The function SumConvergence — 62
3.6.1	Example — 63

3.7	Riemann's theorem on conditionally convergent series — 65
3.8	The Cauchy product of series — 67
3.9	Divergent series — 68
3.10	Power series — 71
4	Limits of functions and continuity — 75
4.1	Limits of functions — 75
4.2	One-sided limits — 76
4.3	Continuous functions — 78
4.4	Discontinuous functions — 79
4.4.1	Example: the Dirichlet function — 82
4.5	The main theorems on continuous functions — 84
4.5.1	Example — 85
4.6	Inverse functions and their continuity — 86
4.7	Example: recursive sequences and continuity — 88
4.8	Uniform continuity and the Lipschitz property — 90
4.8.1	Example — 92
5	Differentiation — 95
5.1	Difference quotient and derivative of a function — 95
5.2	Differentiation in Mathematica® — 102
5.2.1	Differentiation of expressions using D — 102
5.2.2	Differentiation of functions using Derivative — 105
5.2.3	Algebraic rules of differentiation — 107
5.2.4	Example: user-defined derivative — 108
5.3	Main properties of differentiable functions — 109
5.3.1	Example: global and local extrema — 111
5.3.2	Example — 114
5.3.3	Example: the inverse function of the hyperbolic sine — 117
5.4	Convex functions — 119
5.4.1	Jensen's inequality — 123
6	Sequences and series of functions — 125
6.1	Power series continued — 125
6.1.1	Example — 126
6.1.2	Example — 127
6.2	Taylor polynomials and Taylor series — 129
6.2.1	Example — 135
6.2.2	Example — 136
6.2.3	Approximating functions by Taylor polynomials — 137
6.2.4	Example: rational approximation of $\sqrt{e}$ — 139

6.2.5	Example: illustration of approximation of functions with Taylor polynomials —	140
6.3	Convergence of sequences and series of functions —	142
6.3.1	Examples: pointwise, uniform and almost uniform convergence of function sequences —	145
6.3.2	Continuity and differentiability of limits and sums —	149
6.3.3	Examples: pointwise, uniform and almost uniform convergence of function series —	153
6.3.4	Example —	156
7	Integration —	159
7.1	Indefinite integrals —	159
7.2	The Risch algorithm —	163
7.2.1	Differential algebras —	163
7.2.2	Example 1: integration of rational functions —	165
7.2.3	Example 2: the Risch algorithm for an exponential extension —	167
7.2.4	Limitations of Mathematica [®] 's integration —	168
7.3	The Riemann integral —	169
7.3.1	Using Integrate and NIntegrate with definite integrals —	175
7.3.2	Riemann sums —	176
7.4	Improper integrals —	178
7.4.1	Integrals over infinite intervals (improper integrals of the first type) —	178
7.4.2	Improper integrals of the first type and infinite sums —	180
7.4.3	Integrals of unbounded functions (improper integrals of the second type) —	184

Bibliography — 187

Index — 189