

1	Motivation and Prerequisites	1
1.1	Goals	1
1.1.1	Algebraic Structures	1
1.1.2	Polynomial Equations in One Variable	3
1.2	Prerequisites	6
2	Field Extensions and Algebraic Elements	9
2.1	Field Extensions	9
2.2	Intermediate Fields and Algebraic Elements	16
3	Groups	35
3.1	General Definition and Consequences	35
3.2	Subgroups and Group Homomorphisms	42
4	Group Quotients and Normal Subgroups	51
4.1	Equivalence Relations	51
4.2	Group Quotients	58
4.3	The Lagrange Theorem	61
4.4	Normal Subgroups and Factor Groups	62
4.5	The Fundamental Homomorphism Theorem for Groups	68
4.6	Finite Cyclic Groups	70
5	Rings and Ideals	75
5.1	Commutative Rings with Unity	75
5.2	Ring Homomorphisms	78
5.3	Units and Zero Divisors	79
5.4	Ideals, Factor Rings, and the Fundamental Homomorphism Theorem	84
5.5	Prime Ideals and Maximal Ideals	91
5.6	The Chinese Remainder Theorem	95
5.7	Examples of Rings in Quadratic Number Fields	98
6	Euclidean Rings, Principal Ideal Rings, Noetherian Rings	105
6.1	Euclidean Rings	105
6.2	The Euclidean Algorithm	108
6.3	Noetherian Rings	110

7	Unique Factorization Domains	115
7.1	Prime Elements and Irreducible Elements, Unique Factorization Domains	115
7.2	Properties	118
8	Quotient Fields for Domains	123
9	Irreducible Polynomials in UFDs	125
9.1	Content of Polynomials	125
9.2	Reduction Modulo Prime Elements	127
9.3	The Gauss Lemma	128
9.4	Application of the Reduction Mod p	132
10	Galois Theory (I)—Theorem A and Its Variant A'	141
10.1	The Miraculous Creation of some Field	141
10.2	The Splitting Field	144
10.3	Theorem A and A'	151
10.4	Application for the Tower of Field Extensions	157
10.5	The Galois Group	160
11	Intermezzo: An Explicit Example $X^5 - 777X + 7$	169
12	Normal Field Extensions	177
12.1	Algebraic Closure	177
12.2	Extension of Field Homomorphisms	178
12.3	Normal Extensions	179
13	Separability	185
13.1	Motivation and Definition	185
13.2	Formal Derivation	187
13.3	Characteristic of a Field and Separability	192
13.4	The Degree of Separability	194
13.5	The Primitive Element Theorem	199
14	Galois Theory (II)—The Fundamental Theorem	205
14.1	The Fundamental Theorem of Galois Theory—Statement	205
14.2	Outlook on an Application—Quadratic Formula for All Degrees?	206
14.3	Proof of the Fundamental Theorem	213
14.4	Proof of the Addendum	216
15	Cyclotomic Fields	221
15.1	Roots of Unity	221
15.2	Cyclotomic Fields and Cyclotomic Polynomials	226
16	Finite Fields	239
16.1	Prime Fields, Finite Fields, and Frobenius	239
16.2	Finite Fields	241

17	More Group Theory—Group Actions and Sylow's Theorems	253
17.1	Group Actions	253
17.2	The Sylow Theorems	261
17.3	Applications of Sylow's Theorems and Some Common Tricks	263
17.4	Proof of Sylow's Theorems	263
18	Solvability of Polynomial Equations	271
18.1	Solvable Groups.	271
18.2	Solving Polynomial Equations by Radicals.	274
18.3	The General Equation of Degree n	277
	Proof of the Existence of an Algebraic Closure	285
	Tricks and Methods to Classify Groups of a Given Order	291