

Contents

1	Introduction	1
1.1	Multibody Systems	1
1.2	Physical Foundations of Multibody Dynamics	2
1.3	Development of Multibody Dynamics	6
1.4	Multibody Formalisms	8
1.5	Applications of Multibody Systems	10
1.6	Table of Contents	16
2	Basics of Vector Calculus	19
2.1	Scalars and Vectors	19
2.2	Coordinates of Vectors	19
2.3	Calculation Rules for Vectors	21
2.4	Transformation of Vector Coordinates	27
2.5	Second-Order Tensors	30
3	Basics of Kinematics	33
3.1	General Motion of the Rigid Body	33
3.1.1	Task	33
3.1.2	Position	35
3.1.3	Velocity, Twist	36
3.1.4	Rotation Matrix and Angular Velocity	37
3.1.5	Acceleration	38
3.1.6	Example for the Motion of a Rigid Body	38
3.1.7	Special Cases of General Motion	40
3.2	Relative Motions of Rigid Bodies	41
3.2.1	Relative Time Derivative of a Vector	41
3.2.2	Composition of Two Motions	45
3.2.3	Example for the Composition of Two Motions	48
3.2.4	Inverse Relative Motion	52
3.3	Rotation Tensor	55
3.3.1	Axis-Angle Pair	55

3.3.2	RODRIGUES Equation	56
3.3.3	Passive and Active Rotations	58
3.3.4	Axis-Angle Pair from a Given Rotation Matrix	60
3.3.5	Examples of Rotations	60
3.4	Multiple Rotations	62
3.4.1	Definition of Multiple Rotations by Virtual Revolute Joints	63
3.4.2	Rotations Around the Initial Axes	65
3.4.3	Rotations Around the Co-rotating Axes	67
3.5	Rotation Tensor and Angular Velocity	69
3.5.1	Infinitesimal Rotation and Angular Velocity	69
3.5.2	Kinematic Differential Equations of the Rotation Tensor	70
3.5.3	Parameterization of Rotations	73
3.6	CARDAN Angles and EULER Angles	74
3.6.1	xyz CARDAN Angles and Rotation Matrix	75
3.6.2	xyz CARDAN Angles from a Given Rotation Matrix	77
3.6.3	xyz CARDAN Angles and Angular Velocity	78
3.6.4	Comparison of Different Rotation Sequences	80
3.7	EULER Parameters (Quaternions)	80
3.7.1	EULER Parameters and Rotation Tensor	80
3.7.2	EULER Parameters from a Given Rotation Matrix	85
3.7.3	EULER Parameters as Quaternions	86
3.7.4	EULER Parameters and Multiple Rotations	87
3.7.5	EULER Parameters and Angular Velocity	88
3.7.6	Axis-Angle Pair and Angular Velocity	90
3.8	RODRIGUES Vector	93
3.8.1	RODRIGUES Vector and Rotation Tensor	93
3.8.2	RODRIGUES Vector and Angular Velocity	94
3.9	Examples of Parameterization of Rotations	95
3.9.1	Coordinates of a Given Rotation	95
3.9.2	Kinematic Differential Equations	97
4	Basics of Dynamics	101
4.1	Linear and Angular Momentum	101
4.1.1	Linear Momentum	101
4.1.2	Angular Momentum	102
4.2	Center of Mass Theorem and Angular Momentum Law	104
4.2.1	Linear Momentum Law and Center of Mass Theorem	104
4.2.2	Angular Momentum Law	105
4.3	Linear and Angular Momentum of the Rigid Body	107
4.3.1	Mass and Center of Mass	107
4.3.2	Linear Momentum of the Rigid Body	108
4.3.3	Angular Momentum of the Rigid Body	108

4.4	Inertia Tensor	109
4.4.1	Properties of the Inertia Tensor	110
4.4.2	Parallel Displacement of the Reference System	111
4.4.3	Rotation of the Reference System	112
4.4.4	Principal Axis System	113
4.4.5	Inertia Tensor of a Homogeneous Circular Cylinder	114
4.5	Linear and Angular Momentum Laws for the Rigid Body	116
4.5.1	Linear Momentum Law	116
4.5.2	Angular Momentum Law	116
4.5.3	D’ALEMBERT’s Inertial Forces and Torques	118
4.6	Work and Energy	119
4.6.1	Work-Energy Theorem	119
4.6.2	Conservative Forces, Potential Energy, Law of Conservation of Energy	120
4.6.3	Kinetic Energy of the Rigid Body	122
4.7	Dynamic Effects of Rotors	123
4.8	Dynamics of Gyros	126
4.8.1	Torque-free Gyro	126
4.8.2	Heavy Symmetric Gyro	130
5	Holonomic Point Mass Systems	135
5.1	Classification of Constraints	135
5.1.1	Scleronomic and Rheonomic Holonomic Constraints	136
5.1.2	Bilateral and Unilateral Holonomic Constraints	139
5.2	Holonomic Constraints in Point Mass Systems	140
5.2.1	Implicit Holonomic Constraints	140
5.2.2	Degree of Freedom and Minimal Coordinates	142
5.2.3	Explicit Holonomic Constraints	144
5.3	Linear Momentum Laws	147
5.4	Applied Forces	148
5.4.1	Ideal Applied Forces	148
5.4.2	Non-Ideal Friction Forces	151
5.5	Constraint Forces	152
5.5.1	Principle of D’ALEMBERT-LAGRANGE	152
5.5.2	Principle of JOURDAIN	155
5.5.3	Explicit Constraint Reaction Conditions	156
5.5.4	Implicit Constraint Reaction Conditions	157
5.6	Equations of Motion for Point Mass Systems	158
5.6.1	Equations of Motion in Absolute Coordinates	158
5.6.2	Equations of Motion in Minimal Coordinates	161
5.7	Examples of Point Mass Systems	166
5.7.1	Double Point Mass Pendulum	166

5.7.2	Planar Slider-Crank Mechanism	171
5.7.3	Point Mass Sliding on a Rotating Ring	176
5.8	Other Methods of Analytical Dynamics	180
5.8.1	Principle of GAUSS	180
5.8.2	LAGRANGE Equations of the Second Kind	182
5.8.3	Canonical Equations of HAMILTON	186
5.9	On the Numerical Solution of Equations of Motion	189
5.9.1	Ordinary Differential Equations.	189
5.9.2	Differential-Algebraic Equations.	192
5.9.3	Nonlinear Systems of Equations	195
6	Holonomic Planar Multibody Systems	199
6.1	Motion Quantities of Planar Multibody Systems.	199
6.2	Holonomic Constraints in Planar Multibody Systems.	200
6.2.1	Implicit Holonomic Constraints.	200
6.2.2	Explicit Holonomic Constraints.	202
6.3	Linear and Angular Momentum Laws	205
6.4	Applied Forces and Torques.	206
6.5	Constraint Forces and Torques.	208
6.5.1	Principle of JOURDAIN for Planar Multibody Systems	208
6.5.2	Explicit Constraint Reaction Conditions	210
6.5.3	Implicit Constraint Reaction Conditions	210
6.6	Equations of Motion of Planar Multibody Systems.	210
6.6.1	Equations of Motion in Absolute Coordinates.	211
6.6.2	Equations of Motion in Minimal Coordinates	211
6.7	Examples of Planar Multibody Systems	214
6.7.1	Double Pendulum.	214
6.7.2	Rolling Pendulum	221
7	Holonomic Spatial Multibody Systems	225
7.1	Motion Quantities of Spatial Multibody Systems	225
7.2	Holonomic Constraints in Spatial Multibody Systems	227
7.2.1	Implicit Holonomic Constraints.	227
7.2.2	Explicit Holonomic Constraints.	230
7.3	Linear and Angular Momentum Laws	233
7.4	Applied Forces and Torques.	234
7.5	Constraint Forces and Torques.	235
7.5.1	Principle of JOURDAIN for Spatial Multibody Systems	235
7.5.2	Explicit Constraint Reaction Conditions	236
7.5.3	Implicit Constraint Reaction Conditions	236
7.6	Equations of Motion of Spatial Multibody Systems	236
7.6.1	Equations of Motion in Absolute Coordinates.	237
7.6.2	Equations of Motion in Minimal Form	237

7.7	Examples of Spatial Multibody Systems	240
7.7.1	Heavy Gyro with Fixed Point Bearing	240
7.7.2	Robot with Two Revolute Joints	248
8	Nonholonomic Systems	255
8.1	Kinematics of Nonholonomic Systems	255
8.1.1	Implicit Nonholonomic Constraints	256
8.1.2	Degree of Freedom of Nonholonomic Systems	258
8.1.3	Minimal Coordinates and Minimal Velocities	258
8.1.4	Explicit Nonholonomic Constraints	259
8.2	Dynamics of Nonholonomic Systems	260
8.2.1	Equations of Motion in Absolute Coordinates	261
8.2.2	Equations of Motion in Minimal Form	262
8.3	Ball Rolling on a Rotating Plane	265
8.3.1	Equations of Motion in Absolute Coordinates	265
8.3.2	Equations of Motion in Minimal Form	270
8.4	Integrability of Kinematic Constraints	272
8.4.1	Cart with Two Wheels (Knife)	273
8.4.2	Integrability Conditions	276
8.4.3	Applications of Integrability Conditions	279
9	Constraints in Multibody Systems	289
9.1	Joints in Multibody Systems	289
9.1.1	Joints with Holonomic Scleronomic Constraints	290
9.1.2	Joints with Holonomic Rheonomic Constraints	292
9.1.3	Modeling of Joints	292
9.2	Classifications of Multibody Systems	294
9.2.1	Topological Classification	295
9.2.2	Kinematic Classification	295
9.3	Degree of Freedom of Multibody Systems	298
9.3.1	Degree of Freedom of Spatial Multibody Systems	298
9.3.2	Degree of Freedom of Overconstrained Multibody Systems	302
9.3.3	Degree of Freedom of Planar and Spherical Systems	302
9.4	Implicit Holonomic Constraints of Joints	304
9.4.1	Implicit Constraints of a General Joint	304
9.4.2	Elementary Implicit Constraints	306
9.4.3	Implicit Constraints of the Revolute Joint	309
9.5	Explicit Holonomic Constraints of Joints	311
9.5.1	Explicit Constraints of a General Joint	311
9.5.2	Explicit Constraints of the Revolute Joint	315
9.5.3	Explicit Constraints of the Prismatic Joint	316
9.5.4	Explicit Constraints of the Spherical Joint	317

9.6	Constraint Reaction Conditions for Joints	319
9.6.1	Explicit Constraint Reaction Conditions	320
9.6.2	Implicit Constraint Reaction Conditions	320
9.6.3	Constraint Reaction Conditions for the Revolute Joint	321
9.7	Use of Constraints in Motion Equations	322
10	Open Multibody Systems	327
10.1	Topology of Open Multibody Systems	327
10.2	Kinematics of Open Multibody Systems	329
10.2.1	Minimal Coordinates and Minimal Velocities	330
10.2.2	Explicit Constraints in a Chain Structure	331
10.2.3	Chain Structure With Standard Joints	337
10.2.4	Explicit Constraints in a Tree Structure	341
10.3	Dynamics of Open Multibody Systems	342
10.3.1	Implicit Constraint Reaction Conditions for Open Multibody Systems	342
10.3.2	Linear and Angular Momentum Laws	347
10.4	Equations of Motion of Open Multibody Systems	348
10.5	Non-Recursive Formalism for Open Multibody Systems	350
10.6	Recursive Formalism for Open Multibody Systems	352
10.6.1	Recursive Solution for Systems with Chain Structure	353
10.6.2	Recursive Solution for Tree-Structured Systems	357
10.7	Articulated Arm Robot	359
11	Closed Multibody Systems	367
11.1	Kinematics of a Single Multibody Loop	367
11.1.1	Implicit Closure Conditions	368
11.1.2	Explicit Closure Conditions	372
11.1.3	Kinematics of a Planar Four-Bar Mechanism	375
11.2	Kinematics of Multi-Loop Systems	385
11.2.1	Primary Joint Coordinates	385
11.2.2	Implicit Closure Conditions	387
11.2.3	On the Definition of Independent Loops	390
11.2.4	Additional Kinematic Chains	391
11.2.5	Explicit Closure Conditions	392
11.3	Dynamics of Closed Multibody Systems	395
11.3.1	Secondary Constraint Wrenches	395
11.3.2	Linear and Angular Momentum Laws	396
11.4	Equations of Motion in the Primary Joint Coordinates	396
11.4.1	Non-Recursive Formalism for Closed MBS	398
11.4.2	Recursive Formalism for Closed MBS	400
11.5	Equations of Motion in Minimal Coordinates	404
11.5.1	State-Space Equations	405

11.5.2	Minimal Form of the Equations of Motion	406
11.6	Dynamics of a Planar Slider-Crank Mechanism	407
11.6.1	Equations of Motion in Primary Joint Coordinates	407
11.6.2	Equations of Motion in Minimal Form	410
11.7	Dynamics of a Spatial RSRRR Mechanism.	411
11.7.1	Equations of Motion in Primary Joint Coordinates	412
11.7.2	Equations of Motion in Minimal Form	416
11.8	Dynamics of a Parallel Robot	420
11.8.1	Multibody Models Without Redundant Constraints	421
11.8.2	Equations of Motion in Primary Joint Coordinates	423
Mathematical Basics		429
References		437