

Contents

Preface	ix
Suggestions to the Reader	xv
Symbols, Conventions, and Transforms	xvii

Part I: Basic Wavelet Analysis

1. Preliminaries: Background and Notation

1.1 Linear Algebra and Dual Bases	3
1.2 Inner Products, Star Notation, and Reciprocal Bases	12
1.3 Function Spaces and Hilbert Spaces	19
1.4 Fourier Series, Fourier Integrals, and Signal Processing	26
1.5 Appendix: Measures and Integrals	34
Exercises	41

2. Windowed Fourier Transforms

2.1 Motivation and Definition of the WFT.....	44
2.2 Time-Frequency Localization	50
2.3 The Reconstruction Formula	53
2.4 Signal Processing in the Time-Frequency Domain	56
Exercises	58

3. Continuous Wavelet Transforms

3.1 Motivation and Definition of the Wavelet Transform	60
3.2 The Reconstruction Formula	64
3.3 Frequency Localization and Analytic Signals	70
3.4 Wavelets, Probability Distributions, and Detail	73
Exercises	77

4. Generalized Frames: Key to Analysis and Synthesis

4.1 From Resolutions of Unity to Frames 78

4.2 Reconstruction Formula and Consistency Condition 85

4.3 Discussion: Frames versus Bases 88

4.4 Recursive Reconstruction 89

4.5 Discrete Frames 91

4.6 Roots of Unity: A Finite-Dimensional Example 95

Exercises 96

5. Discrete Time-Frequency Analysis and Sampling

5.1 The Shannon Sampling Theorem 99

5.2 Sampling in the Time-Frequency Domain 102

5.3 Time Sampling versus Time-Frequency Sampling 110

5.4 Other Discrete Time-Frequency Subframes 112

5.5 The Question of Orthogonality 119

Exercises 121

6. Discrete Time-Scale Analysis

6.1 Sampling in the Time-Scale Domain 123

6.2 Discrete Frames with Band-Limited Wavelets 124

6.3 Other Discrete Wavelet Frames 133

Exercises 137

7. Multiresolution Analysis

7.1 The Idea 139

7.2 Operational Calculus for Subband Filtering Schemes 147

7.3 The View in the Frequency Domain 160

7.4 Frequency Inversion Symmetry and Complex Structure 169

7.5 Wavelet Packets 172

Exercises 174

8. Daubechies' Orthonormal Wavelet Bases

8.1 Construction of FIR Filters 176

8.2 Recursive Construction of ϕ in the Frequency Domain 183

8.3 Recursion in Time: Diadic Interpolation 189

8.4 The Method of Cumulants191

8.5 Plots of Daubechies Wavelets 197

Exercises 197

Part II: Physical Wavelets

9. Introduction to Wavelet Electromagnetics

9.1 Signals and Waves203

9.2 Fourier Electromagnetics 206

9.3 The Analytic-Signal Transform 215

9.4 Electromagnetic Wavelets 224

9.5 Computing the Wavelets and Reproducing Kernel229

9.6 Atomic Composition of Electromagnetic Waves 233

9.7 Interpretation of the Wavelet Parameters 236

9.8 Conclusions 244

10. Applications to Radar and Scattering

10.1 Ambiguity Functions for Time Signals 245

10.2 The Scattering of Electromagnetic Wavelets256

11. Wavelet Acoustics

11.1 Construction of Acoustic Wavelets271

11.2 Emission and Absorption of Wavelets275

11.3 Nonunitarity and the Rest Frame of the Medium281

11.4 Examples of Acoustic Wavelets 283

References291

Index 297