

Contents

Declaration of Authorship	i
Abstract	iii
Zusammenfassung	v
Acknowledgements	vii
Contents	viii
List of Figures	x
List of Tables	xiii
Abbreviations	xiv
Symbols	xv
1 Introduction	1
2 Finite Element Method	5
2.1 Linear elasticity	5
2.1.1 Governing equations	6
2.1.2 Variational form	7
2.1.3 Discretization	8
2.2 Elastoplasticity	9
2.2.1 Governing equations	9
2.2.2 Flow function	10
2.2.3 Constitutive law	11
2.2.4 Variational form	12
2.2.5 Newton linearization	12
2.2.6 Line search	14
3 Error Estimation	16
3.1 Kelly error estimation	17
3.2 Residual-based error estimation	18

4	Goal-Oriented Error Estimation	23
4.1	Primal problem	23
4.2	Dual problem	24
4.2.1	Elasticity problem	24
4.2.2	Elastoplasticity problem	25
4.3	Quantity of interest functional	27
4.3.1	Displacement at a point	27
4.3.2	Average stress on a surface	27
4.3.3	Average stress in a region	28
5	Mesh adaptivity	30
5.1	Adaptivity criterion	30
5.2	Mesh Adaptivity	31
5.3	General Algorithm	34
5.3.1	Goal-oriented mesh adaptivity in elasticity problems	34
5.3.2	Mesh adaptivity in elastoplasticity problems	35
6	Numerical Examples	37
6.1	Linear Elasticity Problems	38
6.1.1	Plate with a hole under far-field uni-directional tension [1]	38
6.1.2	Point displacement evaluation of a square-shape sample under pressure and shear [1]	41
6.1.2.1	Homogeneous case	42
6.1.2.2	2-materials layered case	42
6.1.2.3	Heterogeneous case	44
6.1.3	Mean stress evaluation on a surface in a square-shape sample with a hole under pressure [1]	46
6.1.3.1	Homogeneous case	48
6.1.3.2	Heterogeneous case	49
6.1.4	3d elasticity problem [2]	52
6.1.4.1	Global mesh adaptivity	53
6.1.4.2	Mesh adaptivity based on Kelly error indicator	54
6.1.4.3	Mesh adaptivity based on residual-based error estimation	54
6.1.4.4	Goal-oriented mesh adaptivity for mean σ_{xx} on Γ_D	54
6.1.4.5	Comparison of convergence rates	55
6.2	Elastoplasticity Problems	56
6.2.1	Thick tube subjected to internal pressure [3]	56
6.2.2	Round perforated strip under enforced displacement [3]	62
6.2.3	Cantiliver beam under surface pressure [3]	65
7	Conclusion	71
A	Random Fields	74
	Bibliography	76