

Contents

List of symbols	viii
----------------------------------	------

1 Introduction

1.1 Background	1
1.1.1 Classical Morse Theory	1
1.1.2 Relative Morse Theory	5
1.1.3 The Continuation Principle	7
1.2 Overview	8
1.2.1 The Construction of the Morse Homology	8
1.2.2 The Axiomatic Approach	10
1.3 Remarks on the Methods	14
1.4 Table of Contents	17
1.5 Acknowledgments	18

2 The Trajectory Spaces

2.1 The Construction of the Trajectory Spaces	21
2.2 Fredholm Theory	29
2.2.1 The Fredholm Operator on the Trivial Bundle	29
2.2.2 The Fredholm Operator on Non-Trivial Bundles	39
2.2.3 Generalization to Fredholm Maps	41
2.3 Transversality	41
2.3.1 The Regularity Conditions	42
2.3.2 The Regularity Results	49

2.4	Compactness	51
2.4.1	The Space of Unparametrized Trajectories	52
2.4.2	The Compactness Result for Unparametrized Trajectories	55
2.4.3	The Compactness Result for Homotopy Trajectories	63
2.4.4	The Compactness Result for λ -Parametrized Trajectories	67
2.5	Gluing	68
2.5.1	Gluing for the Time-Independent Trajectory Spaces	68
2.5.2	Gluing of Trajectories of the Time-Dependent Gradient Flow	96
2.5.3	Gluing for λ -Parametrized Trajectories	99

3 Orientation

3.1	Orientation and Gluing in the Trivial Case	104
3.1.1	The Determinant Bundle	104
3.1.2	Gluing and Orientation for Fredholm Operators	107
3.2	Coherent Orientation	113
3.2.1	Orientation and Gluing on the Manifold M	115

4 Morse Homology Theory

4.1	The Main Theorems of Morse Homology	133
4.1.1	Canonical Orientations	133
4.1.2	The Morse Complex	135
4.1.3	The Canonical Isomorphism	140
4.1.4	Topology and Coherent Orientation	153
4.2	The Eilenberg-Steenrod Axioms	163
4.2.1	Extension of Morse Functions and Induced Morse Functions on Vector Bundles	164
4.2.2	The Homology Functor and Homotopy Invariance	169
4.2.3	Relative Morse Homology	180
4.2.4	Summary	193
4.3	The Uniqueness Result	194

5 Extensions	
5.1 Morse Cohomology	199
5.2 Poincaré Duality	200
5.3 Products	202
A Curve Spaces and Banach Bundles	
A.1 The Manifold of Maps $\mathcal{P}_{x,y}^{1,2}(\mathbb{R}, M)$	207
A.2 Banach Bundles on $\mathcal{P}_{x,y}^{1,2}(\mathbb{R}, M)$	214
B The Geometric Boundary Operator	221
Bibliography	229
Index	232