## **Contents**

1	Intro	Introduction					
2	Con	Continuous Gibrat's Law and Gabaix's Derivation					
	of Zipf's Law						
	2.1	Definition of Continuous Gibrat's Law	9				
	2.2	Geometric Brownian Motion	11				
	2.3	Self-Similar Properties of the Geometric Brownian Motion	12				
	2.4	Time Reversible Geometric Brownian Motion	12				
	2.5	Balance Condition	13				
	2.6	Log-Normal Distribution	14				
	2.7	Gabaix's Steady-State Distribution	16				
3	Flow of Firm Creation						
	3.1	Empirical Evidence and Previous Works					
		on the Arrival of New Firms	19				
	3.2	Mathematical Formulation of the Flow of Firm's					
		Births at Random Instants	21				
	3.3	Existence of a Steady-State Distribution of Firm's Sizes	24				
	3.4	Steady-State Density of Firm's Asset Values Obeying					
		Gibrat's Law	2 <i>6</i>				
	3.5	Mean Density of Firms Younger than Age t	28				
	3.6	Heuristic derivation of the origin of the power law					
		distribution of firm sizes given by Gibrat's rule	29				
4	Usef	'ul Properties of Realizations of the Geometric					
	Brownian Motion						
	4.1	Relationship Between the Distributions of Firm's Mean					
		Ages and Sizes	41				
	4.2	Mean Growth vs. Stochastic Decay	43				
	4.3	Geometrically Transparent Definitions of Stochastically					
		Decaying and Growing Processes	45				
	4.4	Majorant Curves of Stochastically Decaying Geometric					
		Brownian Motion	47				



viii Contents

	4.5	Maximal Value of Stochastically Decaying Geometric	40					
		Brownian Motion	48					
	4.6	Extremal Properties of Realizations of Stochastically						
		Growing Geometric Brownian Motion						
	4.7	Quantile Curves	52					
	4.8	Geometric Explanation of the Steady-State Density						
		of a Firm's Asset Value	22					
5	Exit or "Death" of Firms							
	5.1	Empirical Evidence and Previous Works						
		on the Exit of Firms						
	5.2	Life-Span Above a Given Level						
	5.3	Distribution of Firm's Life Durations Above a Survival Level	62					
	5.4	Killing of Firms upon First Reaching						
		a Given Asset Level from Above						
	5.5	Life-Span of Finitely Living Firms						
	5.6	Influence of Firm's Death on the Balance Condition						
	5.7	Firm's Death Does Not Destroy Zipf's Law						
	5.8	Robustness Vis-a-vis the Randomness of Initial Firm's Sizes	70					
6	Deviations from Gibrat's Law							
	and	Implications for Generalized Zipf's Laws	73					
	6.1	Generalized Brownian Motions	74					
		6.1.1 Statistical Properties of Generalized GBM	74					
		6.1.2 Deterministic Skeleton of the Mean						
		Density $g(s)$ Given by a Generalized-GBM	77					
		6.1.3 Size Dependent Drift and Volatility	78					
	6.2	Diffusion Process with Constant Volatility	79					
	6.3	Steady-State Density of Firm's Asset Values						
		in the Presence of Deviations from Gibrat's Law	82					
	6.4	Integrated Flow	84					
	6.5	The Semi-Geometric Brownian Motion	86					
	6.6	Zipf's Laws When Gibrat's Law Does Not Hold	90					
7	Firm's Sudden Deaths							
-	7.1	Definition of the Survival Function						
	7.2	Exponential Distribution of Sudden Deaths						
	7.3	Implications of the Existence of Sudden Firm						
	7.4	Zipf's Law in the Presence of Sudden Deaths						
	7.5	Explanation of the Generalized Balance Condition						
	7.6	Some Consequences of the Generalized Balance Condition						
	7.7	Zipf's Law as a Universal Law with a Large Basin of Attraction!						
	7.8	Rate of Sudden Death Depending on Firm's Asset Value						
	7.9	Rate of Sudden Depending on Firm's Age						

Contents ix

8	Non-stationary Mean Birth Rate12						
	8.1		ential Growth of Firm's Birth Rate				
	8.2		inistic Skeleton of Zipf's Law				
	8.3		Model of Birth Rate Coupled with the Overall				
		Firm's	Value	125			
	8.4		lization When Both the Initial Firm's Sizes				
		and the	Minimum Firm's Size Grow at Constant Rates	129			
		8.4.1	Formulation of the Model				
		8.4.2	Pdf $f(s;t,\theta)$ of Firm's Size	132			
		8.4.3	Mean Density $g(s,t)$ of Firm Sizes				
		8.4.4	Local Principle				
		8.4.5	Power Law Exponent and Balance Condition				
		8.4.6	Finite Lifetime of the Economy and Transition				
			to the Power Law Regime	137			
	8.5	Time-D	Dependence of the Average Size of the Global				
		Econor	ny of Firms	141			
9	Prop	erties of	the Realization Dependent Distribution of Firm Sizes	147			
	9.1		ion of the Poissonian Distribution of the Number of Firms				
	9.2		Size and Statistical Fluctuation Effects				
		on the l	Empirical Measurement of Zipf's Law	151			
	9.3	Estimat	Estimation of the Distribution of Firm Sizes				
	9.4	Statistical Fluctuations of the Size of the Global					
		Economy Using Characteristic Functions					
10	Future Directions and Conclusions						
	10.1		s and Acquisitions and Spin-offs				
		10.1.1	General Formalism				
		10.1.2	Mergers and Acquisitions and Spin-offs				
			with Brownian Internal Growth	161			
		10.1.3	Mergers and Acquisitions and Spin-offs				
			with GBM for the Internal Growth Process	163			
	10.2	ary of Main Results					
		10.2.1	Importance of Balance Conditions for Zipf's Law				
		10.2.2	Essential Differences with Gabaix (1999)'s				
			Derivation of Zipf's Law	165			
		10.2.3	Robustness of Zipf's Law as an Attractor for				
		10.2	Large Variance of the GMB of Firm's Growth	166			
			Zango rantanov or the Griff of Friday Growth				
Re	ferenc	es		167			
Inc	lex			171			