

Lawrence Conlon
Differentiable Manifolds
A First Course

1993

• Birkhäuser
Boston · Basel · Berlin

TABLE OF CONTENTS

Preface	xi
Acknowledgments	xiii
Chapter 1. Topological Manifolds	1
1.1. Locally Euclidean Spaces	1
1.2. Manifolds	3
1.3. Quotient Constructions and 2-Manifolds	7
1.4. Partitions of Unity	15
1.5. Imbeddings and Immersions	18
1.6. Manifolds with Boundary	20
Chapter 2. Local Theory	25
2.1. Differentiability Classes	25
2.2. Tangent Vectors	27
2.3. Smooth Maps and Their Differentials	34
2.4. Diffeomorphisms and Maps of Constant Rank	38
2.5. Smooth Submanifolds of Euclidean Space	42
2.6. Constructions of Smooth Functions	47
2.7. Smooth Vector Fields	50
2.8. Local Flows	55
2.9. Critical Points and Critical Values	64
Chapter 3. Global Theory	67
3.1. Smooth Manifolds and Mappings	67
3.2. Diffeomorphic Structures	73
3.3. The Tangent Bundle	74
3.4. Cocycles and Geometric Structures	78

3.5. Global Constructions of Smooth Functions	84
3.6. Manifolds with Boundary	87
3.7. Submanifolds	91
3.8. Homotopy and Isotopy	93
3.9. Degree Theory	95
Chapter 4. Flows and Foliation	101
4.1. Complete Vector Fields	101
4.2. The Lie Bracket	107
4.3. Commuting Flows	110
4.4. Foliations	116
Chapter 5. Lie Groups	127
5.1. Basic Definitions and Facts	127
5.2. Lie Subgroups and Subalgebras	136
5.3. Closed Subgroups	139
5.4. Homogeneous Spaces	145
5.5. Principal Bundles	150
Chapter 6. Covectors and 1-Forms	159
6.1. The Space of 1-Forms	159
6.2. Line Integrals	167
6.3. The First Cohomology Space	173
6.4. Some Topological Applications	180
Chapter 7. Multilinear Algebra	189
7.1. Tensor Algebra	189
7.2. Exterior Algebra	199
7.3. Symmetric Algebra	207
7.4. Multilinear Bundle Theory	209
7.5. The Module of Sections	212

Chapter 8. Integration and Cohomology	221
8.1. The Exterior Derivative	221
8.2. Stokes' Theorem and Singular Homology	228
8.3. The Poincaré Lemma	243
8.4. Exact Sequences	249
8.5. Mayer-Vietoris Sequences	252
8.6. Computations of Cohomology	257
8.7. Degree Theory	260
8.8. Poincaré Duality	263
8.9. The de Rham Theorem	268
Chapter 9. Forms and Foliations	277
9.1. The Frobenius Theorem Revisited	277
9.2. The Normal Bundle and Transversality	281
9.3. Closed, Nonsingular 1-Forms	286
Chapter 10. Riemannian Geometry	293
10.1. Connections	294
10.2. Riemannian Manifolds	302
10.3. Gauss Curvature	306
10.4. Complete Riemannian Manifolds	314
10.5. Geodesic Convexity	327
10.6. The Cartan Structure Equations	331
10.7. Riemannian Homogeneous Spaces	344
Appendix A. Vector Fields on Spheres	349
Appendix B. Inverse Function Theorem	353
Appendix C. Ordinary Differential Equations	359
C.1. Existence and uniqueness of solutions	360
C.2. A digression concerning Banach spaces	362

C.3. Smooth dependence on initial conditions	364
C.4. The Linear Case	366
Appendix D. Sard's Theorem	367
Appendix E. de Rham-Čech Theorem	371
E.1. Čech cohomology	371
E.2. The de Rham-Čech complex	375
Bibliography	383
Index	385