

Leonard Bolc Piotr Borowik

Many-Valued Logics

1 Theoretical Foundations

Springer-Verlag

Berlin Heidelberg New York

London Paris Tokyo

Hong Kong Barcelona

Budapest

Contents

1	Preliminaries	1
1.1	Set Operations	1
1.2	Relations	2
1.3	Partial Functions and Functions	4
1.4	Indexed Families of Sets and Generalized Set Operations	5
1.5	Natural Numbers, Countable Sets	5
1.6	Equivalence Relations, Congruences	6
1.7	Orderings	7
1.8	Trees	9
1.9	Inductive Definitions	10
1.10	Abstract Algebras	12
1.11	Logical Matrices	20
2	Many-Valued Propositional Calculi	23
2.1	Remarks on History	23
2.2	The Definition of a Propositional Calculus	25
2.3	Many-Valued Calculi of Lukasiewicz	27
2.4	Finitely Valued Calculi of Lukasiewicz	30
2.4.1	The Formalized Language of Propositional Calculi	30
2.5	Algebraic Characterization of the n -valued Calculi of Lukasiewicz	32
2.5.1	Lattices	32
2.5.2	Quasi-Boolean Algebras and Heyting Algebra	33
2.5.3	Proper Lukasiewicz Algebras	37
2.5.4	The Lukasiewicz Implication	39
2.5.5	Stone Filters in Proper n -valued Lukasiewicz Algebras	41
2.5.6	The Axiom System for the n -valued Propositional Calculus of Lukasiewicz	42
2.6	Many-Valued Calculi of Post	46
2.6.1	Bibliographical Remarks	46
2.6.2	Post Algebras	46
2.6.3	Post Algebra Filters	49
2.6.4	The Axiom System for the n -valued Post Calculus	51
2.6.5	Many-Valued Post Calculi with Several Designated Truth Values	54
2.6.6	Definability of Functors in the n -valued Post Logic	57

3	Survey of Three-Valued Propositional Calculi	6
3.1	The Three-Valued Calculus of Łukasiewicz (L_3)	6
3.2	The Three-Valued Calculus of Bochvar	6
3.3	The Three-Valued Calculus of Finn	6
3.4	The Three-Valued Calculus of Hallden	6
3.5	The Three-Valued Calculus of Åqvist	6
3.6	The Three-Valued Calculi of Segerberg	7
3.7	The Three-Valued Calculus of Pińóg-Rzepecka	7
3.8	The Three-Valued Calculus of Heyting	7
3.9	The Three-Valued Calculus of Kleene	7
3.10	The Three-Valued Calculus of Reichenbach	7
3.11	The Three-Valued Calculus of Slupecki	7
3.12	The Three-Valued Calculus of Sobociński	7
4	Some n-valued Propositional Calculi: A Selection	7
4.1	The Many-Valued Calculus of Slupecki	7
4.2	The Many-Valued Calculus of Sobociński	8
4.3	The Many-Valued Calculi of Gödel	8
4.4	The Many-Valued Calculus C_{nr}	8
5	Intuitionistic Propositional Calculus	9
5.1	The Intuitionistic Propositional Logic in an Axiomatic Setting	9
5.2	The Natural-Deduction Method for the Intuitionistic Propositional Logic	9
5.3	Characterization of the Intuitionistic Propositional Logic in Terms of the Consequence Operator Cn_I	10
5.4	Algebraic Characterization of the Intuitionistic Propositional Logic	10
5.5	Kripke's Semantics for the Intuitionistic Propositional Calculus	10
6	First-Order Predicate Calculus for Many-Valued Logics	10
6.1	The Language of the First-Order Predicate Calculus	10
6.2	Free Variables and Bound Variables	10
6.3	The Rule of Substitution for Individual Variables	10
6.4	Fundamental Semantic Notions	10
6.5	The Many-Valued First-Order Predicate Calculus of Post	11

7	The Method of Finitely Generated Trees in n-valued Logical Calculi	123
7.1	Introductory Remarks	123
7.2	Finitely Generated Trees for n -valued Propositional Calculi	123
7.3	The Existence of Models for the Propositional Calculus	130
7.4	Finitely Generated Trees for n -valued First-Order Predicate Calculi	133
7.5	Finitely Generated Trees for n -valued Quantifiers	137
8	Fuzzy Propositional Calculi	143
8.1	Introductory Remarks	143
8.2	Fuzzy Sets	143
8.3	Syntactic Introduction	144
8.4	Semantic Basis for Fuzzy Propositional Logics	154
8.5	Remarks on the Incompleteness of Fuzzy Propositional Calculi	171
8.6	First-Order Predicate Calculus for Fuzzy Logics	192
8.6.1	Introductory Remarks	192
8.6.2	Generalized Residual Lattices	192
8.6.3	The Language of the Fuzzy First-Order Predicate Calculus	195
8.6.4	Semantic Consequence Operation	199
8.6.5	Syntax of the Fuzzy First-Order Predicate Calculus	202
8.6.6	Syntactic Consequence Operation	203
8.6.7	An Axiom System for the Fuzzy First-Order Predicate Calculus	204
8.6.8	Fuzzy First-Order Theories	206
9	Approximation Logics	209
9.1	Introduction	209
9.2	Rough Sets	209
9.3	Rough Logics with a Chain of Indistinguishability Relations	212
9.3.1	Basic Concepts	212
9.3.2	Approximate Logical Systems	214
9.3.3	Approximation Theories	219
9.4	Approximation Logics with Partially Ordered Sets of Indiscernibility Relations	221
9.4.1	Plain Semi-Post Algebras	221
9.4.2	Approximation Logic of Type T	225
9.4.3	Approximation Logics of Type T with Many Indiscernibility Relations	228

10	Probability Logics	23
10.1	Introduction	23
10.2	Lukasiewicz' Idea of Logical Probability	23
10.3	An Algebraic Description of Probability Logic	23
10.3.1	Syntax	23
10.3.2	Semantics	23
10.3.3	Constructions	23
10.3.4	Probabilistic Consequence	23
10.4	Axiomatic Approach to Probability Logic	24
10.4.1	Syntax	24
10.4.2	Probability and Probabilistic Consequence	24
10.4.3	Completeness of Probability Logic	24
10.4.4	Applications	25
10.4.5	Unreasonable Inference	25
	References	25
	Index of Symbols	28
	Author Index	28
	Subject Index	28