

Contents

1	A Short Introduction to Conformal Invariance	1
	Malte Henkel and Dragi Karevski	
1.1	From Scale-Invariance to Conformal Invariance	1
1.2	The Energy-Momentum Tensor	7
1.3	The Virasoro Algebra	15
1.4	Kac Formula and Unitary Minimal Models	18
1.5	From Characters to Modular Invariance	21
1.6	The Free Boson	27
1.7	The Sugawara Construction	30
1.8	Compactifications and Modular Invariance	34
1.9	From the Screened Coulomb Gas to Minimal Models	36
1.10	Surface Critical Phenomena	38
1.11	Boundary Conformal Field-Theory	40
1.12	Notes and References	46
	References	47
2	A Short Introduction to Critical Interfaces in 2D	51
	Michel Bauer	
2.1	Introduction	51
2.2	Discrete Models	53
2.2.1	Discrete Domains	54
2.2.2	The Exploration Process and Percolation	57
2.2.3	Loop-Erased Random Walks	66
2.2.4	Another Example of Growth: DLA	75
2.3	Loewner Chains	76
2.3.1	Around Riemann's Theorem	77
2.3.2	Hulls	79
2.3.3	Basic Examples	80
2.3.4	Iteration of Conformal Maps	81
2.3.5	Continuous Time Growth Processes	81
2.3.6	Geometric Interpretation	83

2.3.7	Local Growth	84
2.4	Stochastic Loewner Evolutions	87
2.4.1	Conformal Invariance	88
2.4.2	Domain Markov Property	89
2.4.3	Schramm's Argument	90
2.4.4	Basic Properties	91
2.4.5	Locality	93
2.5	Relation with Conformal Field-Theory	96
2.5.1	Motivation	96
2.5.2	A Crash Course in Boundary CFT (BCFT)	96
2.5.3	Martingales and Singular Vectors	98
2.5.4	Two Examples	99
2.5.5	Conformal SLE Martingales	104
2.6	Notes and References	108
	References	109
3	Numerical Tests of Schramm-Loewner Evolution in Random Lattice Spin Models	113
	Christophe Chatelain	
3.1	Introduction	113
3.2	Genesis and Localisation of the Interface	114
3.3	Numerical Tests of SLE Properties	121
3.3.1	Domain Markov Property	121
3.3.2	Conformal Invariance	122
3.3.3	Driving Function	124
3.3.4	Finite-Size Scaling of Length and Winding Number	127
3.3.5	Left-Passage Probability	128
3.4	SLE in Random Lattice Spin Models	130
3.4.1	Random Potts Model	131
3.4.2	SOS Model on a Random Substrate	133
3.4.3	Ising Spin Glasses	133
3.4.4	Random-Field Ising Model	136
	References	138
4	Loop Models and Boundary CFT	141
	Jesper Lykke Jacobsen	
4.1	Models and Transformations	141
4.1.1	Potts Model	141
4.1.2	$O(n)$ Model	145
4.1.3	Vertex and Height Models	147
4.2	Coulomb Gas in the Bulk	151
4.2.1	Compactified Free Boson	152
4.2.2	Liouville Field-Theory	153
4.2.3	Marginality Requirement	155
4.2.4	Critical Exponents	156
4.3	Coulomb Gas at the Boundary	159

4.3.1	Surface Critical Behaviour	160
4.3.2	Sketch of Boundary CFT	161
4.3.3	Coulomb Gas for Loops on an Annulus	164
4.3.4	BCFT and the $O(n)$ Model	165
4.3.5	Critical Exponents	166
4.4	Temperley-Lieb Algebra	167
4.4.1	Usual TL Algebra	168
4.4.2	One-Boundary TL Algebra	173
4.5	Exact CFT Partition Functions	175
4.5.1	0BTL Loop Model	175
4.5.2	A Percolation Crossing Formula	177
4.5.3	1BTL Loop Model	178
4.5.4	2BTL Loop Model	180
4.6	Notes and References	181
	References	181
	Index	185