

# CONTENTS

## Introduction

vii

## Chapter 0: Galois theory of commutative rings

|    |  |    |
|----|--|----|
| §1 | Definitions and basic properties         | 1  |
| §2 | The main theorem of Galois theory        | 6  |
| §3 | Functoriality and the Harrison product   | 8  |
| §4 | Ramification                             | 17 |
| §5 | Kummer theory and Artin-Schreier theory  | 19 |
| §6 | Normal bases and Galois module structure | 25 |
| §7 | Galois descent                           | 28 |
| §8 | $\mathbb{Z}_p$ -extensions               | 30 |

## Chapter I: Cyclotomic descent

|    |                                       |    |
|----|---------------------------------------|----|
| §1 | Cyclotomic extensions                 | 32 |
| §2 | Descent of normal bases               | 38 |
| §3 | Cyclotomic descent: the main theorems | 45 |

## Chapter II: Corestriction and "Hilbert's Theorem 90"

|    |   |    |
|----|---|----|
| §1 | Corestriction   | 55 |
| §2 | Lemmas on group cohomology                              | 60 |
| §3 | "Hilbert 90": the kernel and image of the corestriction | 62 |
| §4 | Lifting theorems  | 64 |

## Chapter III: Calculations with units

|    |                                       |    |
|----|---------------------------------------|----|
| §1 | Results on twisted Galois modules     | 67 |
| §2 | Finite fields and $\ell$ -adic fields | 70 |
| §3 | Number fields                         | 73 |

## Chapter IV: Cyclic $p$ -extensions and $\mathbb{Z}_p$ -extensions of number fields

|    |   |    |
|----|---|----|
| §1 | $C_{p^n}$ -extensions and ramification                          | 77 |
| §2 | $\mathbb{Z}_p$ -extensions                                      | 79 |
| §3 | The asymptotic order of $P(R, C_{p^n})$                         | 83 |
| §4 | Calculation of $q_k$ : examples                                 | 88 |
| §5 | Torsion points on abelian varieties with complex multiplication | 91 |
| §6 | Further results: a short survey                                 | 95 |

**Chapter V: Geometric theory: Cyclic extensions of finitely generated fields**

|    |  |     |
|----|--|-----|
| §1 | Geometric prerequisites  | 97  |
| §2 | $\mathbb{Z}_p$ -extensions of absolutely finitely generated fields | 101 |
| §3 | A finiteness result  | 106 |

**Chapter VI: Cyclic Galois theory without the condition " $p^{-1} \in R$ "**

|    |  |     |
|----|--|-----|
| §1 | Witt rings and Artin-Schreier theory for rings of characteristic $p$ | 109 |
| §2 | Patching results   | 113 |
| §3 | Kummer theory without the condition " $p^{-1} \in R$ "               | 116 |
| §4 | The main result and Artin-Hasse exponentials                         | 120 |
| §5 | Proofs and examples  | 126 |
| §6 | Application: Generic Galois extensions                               | 135 |

|                   |     |
|-------------------|-----|
| <b>References</b> | 140 |
|-------------------|-----|

|              |     |
|--------------|-----|
| <b>Index</b> | 144 |
|--------------|-----|