

Contents

Introduction	1
Chapter I. Metrically Transitive Operators	9
1 Basic Definitions and Examples	9
1.A Random Variables, Functions and Fields	9
1.B Random Vectors and Operators	13
1.C Metrically Transitive Random Fields	17
1.D Metrically Transitive Operators	33
2 Simple Spectral Properties of Metrically Transitive Operators	36
2.A Deficiency Indices	36
2.B Nonrandomness of the Spectrum and of its Components	41
2.C Nonrandomness of Multiplicities	47
Problems	51
Chapter II. Asymptotic Properties of Metrically Transitive Matrix and Differential Operators	58
3 Review of Basic Results	58
4 Matrix Operators on $\ell_2(\mathbf{Z}^d)$	67
4.A Essential Self-Adjointness	67
4.B Existence of the Integrated Density of States and Other Ergodic Properties	71
4.C Simple Properties of the Integrated Density of States and of the Spectra of Metrically Transitive Matrix Operators	81
4.D Location of the Spectrum	85
5 Schrödinger Operators and Elliptic Differential Operators on $L_2(\mathbf{R}^d)$	87
5.A Criteria for Essential Self-Adjointness	87
5.B Ergodic Properties	98
5.C Some Properties of the Integrated Density of States	108
5.D Location of the Spectrum of a Metrically Transitive Schrödinger Operator	116
Problems	122

Chapter III. Integrated Density of States in One-Dimensional Problems of Second Order	136
6 The Oscillation Theorem and the Integrated Density of States	136
6.A The Phase and the Existence of the Integrated Density of States ...	136
6.B Simplest Asymptotics of the Integrated Density of States at the Edges of the Spectrum	140
6.C Schrödinger Operator with Markov Potential	149
6.D The Brownian Motion Model	156
6.E Jacobi Matrices with Independent and Markov Coefficients	161
6.F Smoothness of $N(\lambda)$; Special Energies	164
7 Examples of Calculation of the Integrated Density of States	172
7.A The Kronig-Penny Stochastic Model	172
7.B Random Jacobi Matrices	176
Problems	180
Chapter IV. Asymptotic Behavior of the Integrated Density of States at Spectral Boundaries in Multidimensional Problems	190
8 Stable Boundaries	190
9 Fluctuation Boundaries: General Discussion and Classical Asymptotics	197
9.A Introduction and Heuristic Discussion	197
9.B Simplest Bounds. Gaussian and Negative Poisson Potentials	202
9.C Generalized Poisson Potential	212
10 Fluctuation Boundaries: Quantum Asymptotics	216
10.A The Lifshitz Exponent	217
10.B Generalized Poisson Potential with a Nonnegative, Rapidly Decreasing Function	221
10.C Smoothed Square of a Gaussian Random Field	236
Problems	250
Chapter V. Lyapunov Exponents and the Spectrum in One Dimension	255
11 Existence and Properties of Lyapunov Exponents	256
11.A The Multiplicative Ergodic Theorem and the Existence of Lyapunov Exponents	256
11.B The Lyapunov Exponent and the Integrated Density of States	264
11.C Simplest Asymptotic Formulas and Estimates for Lyapunov Exponents	278
12 Lyapunov Exponents and the Absolutely Continuous Spectrum	288
12.A Basic Facts About the Spectrum of One-Dimensional Operators of the Second Order	289

12.B Lyapunov Exponents and the Absolutely Continuous Spectrum ...	297
12.C Multiplicity of the Spectrum	306
12.D Deterministic Potentials	307
12.E Some Inverse Problems	312
13 Lyapunov Exponents and the Point Spectrum	314
13.A Heuristic Discussion	314
13.B Conditions for Positive Lyapunov Exponents to Imply a Pure Point Spectrum	317
Problems	329
Chapter VI. Random Operators	342
14 The Lyapunov Exponent of Random Operators in One Dimension	343
14.A Positiveness of the Lyapunov Exponent	344
14.B Asymptotic Formulas for the Lyapunov Exponent	354
15 The Point Spectrum of Random Operators	362
15.A The Pure Point Spectrum in One Dimension	362
15.B Other One-Dimensional Results	378
15.C The Point Spectrum in Multidimensional Problems	387
Problems	403
Chapter VII. Almost-Periodic Operators	415
16 Smooth Quasi-Periodic Potentials	417
16.A The Integrated Density of States and the Gap Labeling Theorem ..	419
16.B Absolutely Continuous Spectrum	428
16.C Lower Bounds of Solutions and Absence of a Point Spectrum	439
16.D Lower Bounds for the Lyapunov Exponent and Absence of an Absolutely Continuous Spectrum in the Discrete Case	445
16.E Point Spectrum of Almost-Periodic Operators	451
16.F The Almost-Mathieu Operator	459
17 Limit-Periodic Potentials	463
17.A Basic Results	465
17.B Spectral Data for Periodic Potentials of Increasing Period	471
17.C Proof of the Main Theorems	493
18 Unbounded Quasiperiodic Potentials	500
18.A General Results and the Integrated Density of States	501
18.B The Case of Strongly Incommensurate Frequencies	505
18.C The One-Dimensional Case	517
18.D The Schrödinger Operator with a Nonlocal Quasiperiodic Potential	527
Problems	534

Appendix A: Nevanlinna Functions 543

Appendix B: Distribution of Eigenvalues of Large Random Matrices 547

Bibliography 558

List of Symbols 584

Index 585