

Lecture Notes in Engineering

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Current Distributions
and Electrode Shape Changes
in Electrochemical Systems



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