Table of Contents

| Chapter XIV. | Evolution | Problems: | Cauchy | Problems | in | R" |
|--------------|-----------|-----------|--------|----------|----|----|
|--------------|-----------|-----------|--------|----------|----|----|

| Intr | oduction | 1 |
|------------|--|---------|
| §1. | The Ordinary Cauchy Problems in Finite Dimensional Spaces | 3 |
| | Linear Systems with Constant Coefficients Linear Systems with Non Constant Coefficients | 4 6 |
| 6.3 | - | |
| g2. | Diffusion Equations | 8 |
| | Setting of Problem | 9 10 |
| | 3. The Elementary Solution of the Heat Equation | 15 |
| | 4. Mathematical Properties of the Elementary Solution and the | 13 |
| | Semigroup Associated with the Heat Operator | 16 |
| §3. | Wave Equations | 21 |
| 0 | 1. Model Problem: The Wave Equation in \mathbb{R}^n | 21 |
| | 2. The Euler-Poisson-Darboux Equation | 44 |
| | 3. An Application of §2 and 3: Viscoelasticity | 48 |
| §4. | The Cauchy Problem for the Schrödinger Equation, Introduction | 53 |
| | 1. Model Problem 1. The Case of Zero Potential | 53 |
| | 2. Model Problem 2. The Case of a Harmonic Oscillator | 57 |
| §5. | The Cauchy Problem for Evolution Equations Related to Convolution | |
| | Products | 58 |
| | 1. Setting of Problem | 58 |
| | 2. The Method of the Fourier Transform | 59 |
| | 3. The Dirac Equation for a Free Particle | 63 |
| §6. | An Abstract Cauchy Problem. Ovsyannikov's Theorem | 66 |
| Rev | iew of Chapter XIV | 72 |
| | | |
| Cha | pter XV. Evolution Problems: The Method of Diagonalisation | |
| Cila | pres Av. Evolution Fromenis. The Method of Diagonalisation | |
| Intr | oduction | 73 |
| §1. | The Fourier Method or the Method of Diagonalisation | 74 |
| | 1. The Case of the Space \mathbb{R}^1 $(n = 1)$ | 74 |
| | 2. The Case of Space Dimension $n = 2 \dots \dots \dots$ | 94 |

| | 3. The Case of Arbitrary Dimension n | 99 103 |
|-------------|--|-------------------|
| §2. | Variations. The Method of Diagonalisation for an Operator Having Continuous Spectrum | 104 |
| | Review of Self-Adjoint Operators in Hilbert Spaces General Formulation of the Problem A Simple Example of the Problem with Continuous Spectrum | 104 104 108 |
| 0.3 | | |
| §3. | Examples of Application: The Diffusion Equation | 112 |
| | for Neutrons | 112 118 122 |
| §4. | The Wave Equation: Mathematical Examples and Examples of | |
| 3 | Application | 126 |
| | 1. The Case of Dimension $n = 1, \dots, \dots$ | 126 |
| | 2. The Case of Arbitrary Dimension n | 143 |
| | 3. Examples of Applications for $n = 1 \dots \dots \dots \dots$ | 145 |
| | 4. Examples of Applications for $n = 2$. Vibrating Membranes | 156 |
| | 5. Application to Elasticity; the Dynamics of Thin Homogeneous Beams | 159 |
| 8 5 | The Schrödinger Equation | 169 |
| g э. | 1. The Cauchy Problem for the Schrödinger Equation in a Domain | |
| | $\Omega =]0,1[\subset \mathbb{R}.$ | 170 |
| | 2. A Harmonic Oscillator | 177 183 |
| §6. | Application with an Operator Having a Continuous Spectrum: Example | 184 |
| - | view of Chapter XV | 186 |
| | pendix. Return to the Problem of Vibrating Strings | 186 |
| AP. | pendix. Return to the Problem of Vibrating Strings | 100 |
| Ch | apter XVI. Evolution Problems: The Method of the Laplace Transform | |
| Int | roduction | 202 |
| § 1. | Laplace Transform of Distributions | 203 |
| 3 | 1. Study of the Set I_{ℓ} and Definition of the Laplace Transform | 204 |
| | 2. Properties of the Laplace Transform | 210 |
| | 3. Characterisation of Laplace Transforms of Distributions of $L_+(\mathbb{R})$. | 212 |
| §2. | Laplace Transform of Vector-valued Distributions | 21 |
| - | 1. Distributions with Vector-valued Values | 218 |
| | 2. Fourier and Laplace Transforms of Vector-valued Distributions. | 222 |

Table of Contents XI

| §3. | Applications to First Order Evolution Problems | 225 |
|--------------|--|------------|
| | 1. 'Vector-valued Distribution' Solutions of an Evolution Equation | |
| | of First Order in t | 225 |
| | 2. The Method of Transposition | 231 |
| | 3. Application to First Order Evolution Equations. The Hilbert Space | |
| | Case. L ² Solutions in Hilbert Space | 233 |
| | 4. The Case where A is Defined by a Sesquilinear Form $a(u, v)$ | 243 |
| §4 . | Evolution Problems of Second Order in t | 251 |
| | 1. Direct Method | 251 |
| | 2. Use of Symbolic Calculus | 257 |
| | Review | 261 |
| §5. | Applications | 261 |
| | 1. Hydrodynamical Problems | 261 |
| | 2. A Problem of the Kinetics of Neutron Diffusion | 265 |
| | 3. Problems of Diffusion of an Electromagnetic Wave | 267 |
| | 4. Problems of Wave Propagation | 273 |
| | 5. Viscoelastic Problems | 280 |
| | 6. A Problem Related to the Schrödinger Equation | 290 |
| | 7. A Problem Related to Causality, Analyticity and Dispersion Relations | 292 |
| | 8. Remark 10 | 295 |
| Re | view of Chapter XVI | 296 |
| | | |
| Ch | apter XVII. Evolution Problems: The Method of Semigroups | |
| CII | upter XVII. Evolution 1 robicins. The Method of Semigroups | |
| Int | roduction | 297 |
| Par | rt A. Study of Semigroups | 301 |
| §1. | Definitions and Properties of Semigroups Acting in a Banach Space. | 301 |
| | 1. Definition of a Semigroup of Class \mathscr{C}^0 (Resp. of a Group) | 301 |
| | 2. Basic Properties of Semigroups of Class \mathscr{C}^0 | 307 |
| \$2. | The Infinitesimal Generator of a Semigroup | 310 |
| 0 | 1. Examples | 310 |
| | 2. The Infinitesimal Generator of a Semigroup of Class \mathscr{C}^0 | 315 |
| 83. | The Hille-Yosida Theorem | 321 |
| 3 | 1. A Necessary Condition | 321 |
| | 2. The Hille-Yosida Theorem | 323 |
| | 3. Examples of Application of the Hille-Yosida Theorem | 327 |
| 8.4 | | 353 |
| § 4 . | | 223 |
| | 1. The Characterisation of the Infinitesimal Generator of a Group of Class & 0 | 252 |
| 1 | | 353 356 |
| | 2. Unitary Groups of Class \mathscr{C}^0 . Stone's Theorem | 330 |

XII Table of Contents

| | Applications of Stone's Theorem Conservative Operators and Isometric Semigroups in Hilbert Space Review | 357 362 365 |
|-----|--|-------------------|
| §5. | Differentiable Semigroups | 365 |
| §6. | Holomorphic Semigroups | 367 |
| §7. | Compact Semigroups | 388 |
| | 1. Definition and Principal Properties | 388 |
| | 2. Characterisation of Compact Semigroups | 389 |
| | 3. Examples of Compact Semigroups | 39.4 |
| Par | t B. Cauchy Problems and Semigroups | 397 |
| §1. | Cauchy Problems | 397 |
| §2. | Asymptotic Behaviour of Solutions as $t \to +\infty$. Conservation and | 40.0 |
| | Dissipation in Evolution Equations | 406 |
| | Semigroups and Diffusion Problems | 412 |
| §4. | Groups and Evolution Equations | 420 |
| | Wave Problems Schrödinger Type Problems | 420 424 |
| | 3. Weak Asymptotic Behaviour, for $t \to \pm \infty$, of Solutions of | 121 |
| | Wave Type of Schrödinger Type Problems | 426 |
| | 4. The Cauchy Problem for Maxwell's Equations in an Open Set $\Omega \subset \mathbb{R}^3 \ldots \ldots \ldots \ldots \ldots \ldots$ | 433 |
| §5. | Evolution Operators in Quantum Physics. The Liouville-von Neumann | |
| Ü | Equation | 439 |
| | Existence and Uniqueness of the Solution of the Cauchy Problem for the Liouville-von Neumann Equation in the Space of Trace Operators The Evolution Equation of (Bounded) Observables in the Heisenberg | 439 |
| | Representation | 446 451 |
| 6.6 | 3. Spectrum and Resolvent of the Operator h | 453 |
| §6. | 1. Convergence of Semigroups | 453 |
| | 2. General Representation Theorem | 459 |
| Sur | mmary of Chapter XVII | 465 |
| Ch | apter XVIII. Evolution Problems: Variational Methods | |
| Int | roduction. Orientation | 467 |
| §1. | Some Elements of Functional Analysis | 469 |
| | 1. Review of Vector-valued Distributions | 469 |
| | 2. The Space $W(a, b; V, V')$ | 472 |

| § 2. | The Spaces W(a, b; X, Y). Extension to Banach Space Framework. An Intermediate Derivatives Theorem Bidual. Reflexivity. Weak Convergence and Weak * Convergence Galerkin Approximation of a Hilbert Space | 479 482 493 499 503 |
|-------------|--|---|
| | 1. Definition | 504 504 507 |
| §3. | Evolution Problems of First Order in t | 509 509 512 513 520 521 |
| §4. | Problems of First Order in t (Examples). Mathematical Example 1. Dirichlet Boundary Conditions. Mathematical Example 2. Neumann Boundary Conditions. Mathematical Example 3. Mixed Dirichlet-Neumann Boundary Conditions. Mathematical Example 4. Bilinear Form Depending on Time t. Evolution, Positivity and 'Maximum' of Solutions of Diffusion Equations in L^p(Ω), 1 ≤ p ≤ ∞. Mathematical Example 5. A Problem of Oblique Derivatives. Example of Application. The Neutron Diffusion Equation. A Stability Result. | 524 524 527 528 533 539 542 |
| §5. | Evolution Problems of Second Order in t | 552 558 561 566 |
| §6. | Problems of Second Order in t. Examples | 581 582 583 587 |
| §7. | Other Types of Equation | 620 643 651 |

| 5. The Problem of Coupled Parabolic-Hyperbolic Transmission 66. The Method of 'Extension with Respect to a Parameter' 6 | 70 76 |
|---|----------|
| Review of Chapter XVIII | |
| Bibliography | 80 |
| Table of Notations | 86 |
| Index | 02 |
| Contents of Volumes 1–4, 6 | 05 |